

# Timescale of magnetic fluctuations across and above the Mott transition

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 DMFT on the Bethe lattice  
 Magnetic susceptibility

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 Adiabatic spin response

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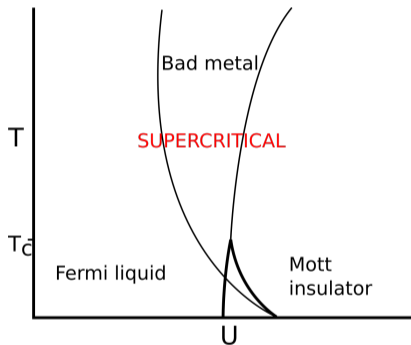


Figure 1 – Cartoon picture of the DMFT phase diagram of the half-filled Hubbard model

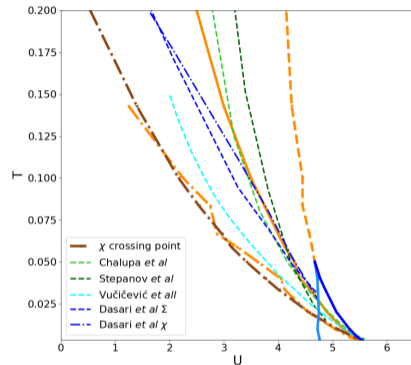


Figure 2 – DMFT phase diagram of the half-filled Hubbard model using previously defined crossover lines [1-5]

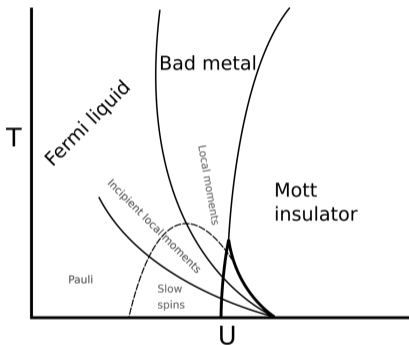


Figure 3 – Cartoon picture of the DMFT phase diagram of the half-filled Hubbard model

We propose a **new characterization** of the phase diagram using only **timescales of magnetic fluctuations** and **valence fluctuations** that brings :

- Physical insight to the crossover lines in the supercritical region
- New "slow spin" dome in the Fermi liquid regime.

Gaspard, L. ; Tomczak, J. M.

Timescale of local moment screening across and above the Mott transition, 2021

<https://arxiv.org/abs/2112.02881v1>

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- One orbital per site ( $i$ )
- Half-filling (one electron per site  $i$ )
- $t$  : nearest neighbor hopping
- $U$  : Coulomb repulsion between electrons on the same site
- $Z = \infty$  Bethe lattice with the density of states :

$$D(\varepsilon) = \frac{\sqrt{4t^2 - \varepsilon^2}}{2\pi t^2}$$

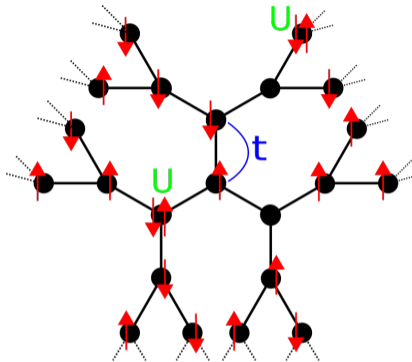


Figure 4 – Schematic picture of the Hubbard model on a  $Z=3$  Bethe lattice

$$\hat{H} = -t \sum_{\langle i,j \rangle, \sigma} \hat{c}_{i,\sigma}^\dagger \hat{c}_{j,\sigma} + U \sum_i \hat{n}_{i,\uparrow} \hat{n}_{i,\downarrow} \quad (1)$$

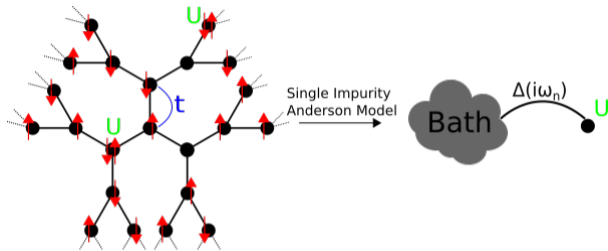


Figure 5 – Schematic picture of the Single Impurity Anderson Model (SIAM)

We replace the Bethe lattice by the SIAM :

$$\hat{H} = \hat{H}_{\text{atom}} + \hat{H}_{\text{bath}} + \hat{H}_{\text{hybridization}}$$

$$\hat{H} = U\hat{n}_{\uparrow}\hat{n}_{\downarrow} + (\varepsilon_0 - \mu)(\hat{n}_{\uparrow} + \hat{n}_{\downarrow}) + \sum_{l,\sigma} \varepsilon_l \hat{a}_{l,\sigma}^{\dagger} \hat{a}_{l,\sigma} + \sum_{l,\sigma} V_l (\hat{a}_{l,\sigma}^{\dagger} \hat{c}_{\sigma} + \hat{c}_{\sigma}^{\dagger} \hat{a}_{l,\sigma})$$



DMFT on the Bethe lattice ( $Z=\infty$ ) is **exact** :

$$\begin{array}{ccc}
 & \Sigma_{\text{imp}}(\omega) & \\
 \text{CT-HYB} \nearrow & & \searrow \\
 \Delta(\omega) = t^2 G_{\text{imp}}(\omega) & \xleftarrow{G_{\text{imp}}=G_{\text{loc}}} & G_{\text{loc}}(\omega) = \int d\varepsilon \frac{1}{\omega - \varepsilon - \Sigma_{\text{imp}}(\omega)} D(\varepsilon)
 \end{array}$$

CT-HYB : Expansion of the partition function in the interaction representation and Monte-Carlo integration

$$Z = \text{Tr} \left[ \mathcal{T}_\tau \exp \left( -\beta (\hat{H}_{\text{atom}} + \hat{H}_{\text{bath}}) \right) \exp \left( - \int_0^\beta d\tau \hat{H}_{\text{hybridization}}(\tau) \right) \right]$$

We are interested in the **local magnetic susceptibility** computed in imaginary time  $\tau$  :

$$\chi_m(\tau) = g^2 \langle \mathcal{T}_\tau \hat{S}_z(\tau) \hat{S}_z(0) \rangle \quad (2)$$

From this we can get the **static local magnetic susceptibility** :

$$\chi_m(i\omega = 0) = \int_0^\beta \chi_m(\tau) d\tau \quad (3)$$

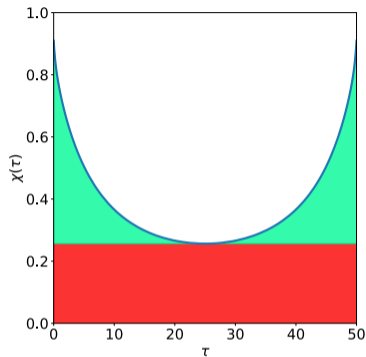


Figure 6 – Local magnetic susceptibility in imaginary time for  $U=4.9$  and  $\beta = 50$

$$\chi_m(i\omega = 0) = \beta \chi_m \left( \tau = \frac{\beta}{2} \right) + \int_0^\beta \chi_m(\tau) - \chi_m \left( \tau = \frac{\beta}{2} \right) d\tau \quad (4)$$

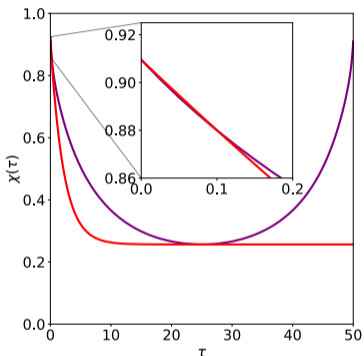


Figure 7 – Local magnetic susceptibility for  $U=4.9$  and  $\beta = 50$

Fit of  $\chi(\tau)$  for  $\tau$  close to 0 :

$$\chi_m \left( \tau \ll \frac{\beta}{2} \right) = \chi_m \left( \tau = \frac{\beta}{2} \right) + \left[ \chi_m(\tau = 0) - \chi_m \left( \tau = \frac{\beta}{2} \right) \right] e^{-\frac{\tau}{t_m}} \quad (5)$$

$t_m$  is the timescale of the magnetic fluctuations

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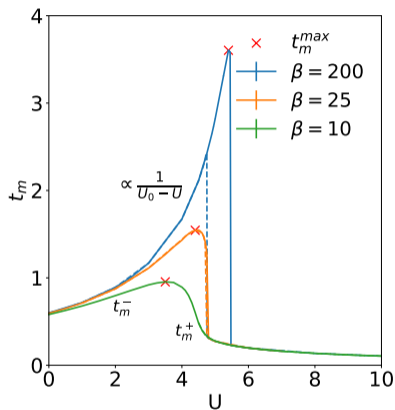
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- At low temperature (**high**  $\beta$ ) : discontinuity in  $t_m$  at the phase transition.
- At high temperature (**low**  $\beta$ ) : continuous evolution of  $t_m$
- In all cases :  $t_m$  reaches a maximum for a value of  $U$

Figure 8 –  $t_m$  as a function of the interaction  $U$  for three values of  $\beta$

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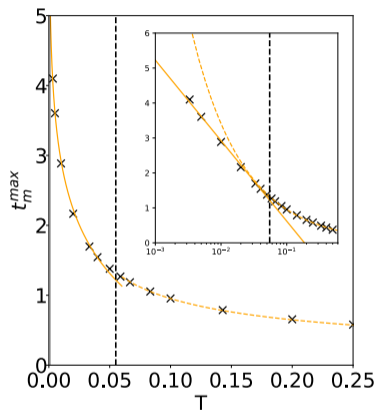


Figure 9 – Value of the maximum of  $t_m$  at constant  $U$  as a function of the temperature

$t_m^{max}(T)$  changes behavior for a given temperature  $T_c \approx 0.055$

- **Below  $T_c$  :**  
 $t_m^{max}$  exhibits a logarithmic behavior :  $t_m^{max} \approx -\log\left(\frac{T}{\gamma}\right)$ ,  
 $\gamma \approx 0.19$
- **Above  $T_c$  :**  
 $t_m^{max}$  behaves as a power :  
 $t_m^{max} \propto T^{-\alpha}$ ,  $\alpha \approx 0.27$

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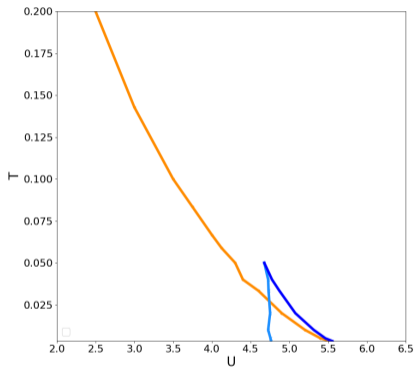


Figure 10 – Phase diagram of the Hubbard model using previously defined quantities

- The coexistence region is reproduced using the **discontinuity in  $t_m$**
- The orange line is the coordinate of the **maximum** of  $t_m$  ( $t_m^{max}$ ). It separates the Fermi liquid (where the magnetic fluctuations timescale increases with the effective mass of the quasiparticle) and the Bad metal (where the magnetic fluctuations become slower).

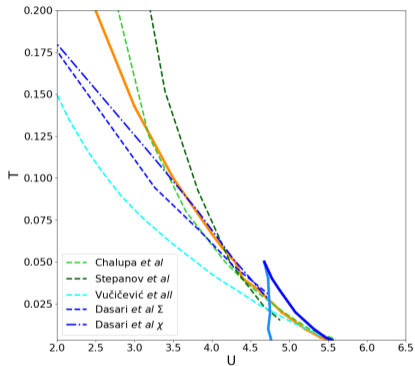


Figure 11 – Comparison the  $t_m^{max}$  line from our work with previous works

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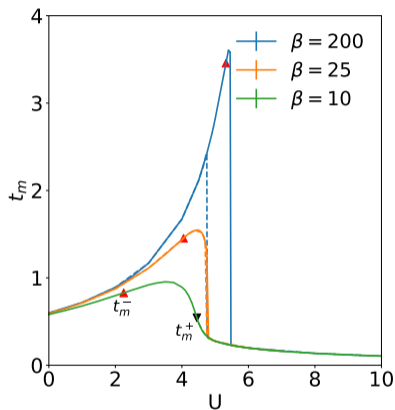


Figure 12 –  $t_m$  as a function of the interaction  $U$  for three values of  $\beta$

- First inflection point of  $t_m$  ( $\blacktriangle$ ), exists in all the cases : formation of a local moment.
- Second inflection point of  $t_m$  ( $\blacktriangledown$ ), exists only when  $t_m$  is continuous : reaching the atomic limit.

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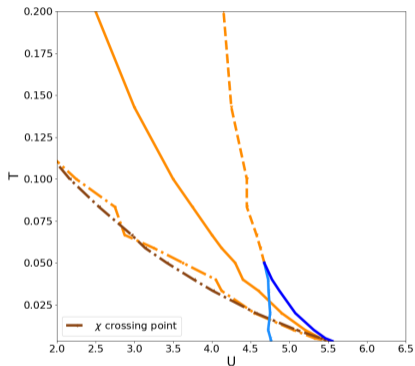


Figure 13 – Comparison the  $t_m^{max}$  line from our work with previous works

- The  $t_m^-$  line is consistent with the crossover line between  $\chi^{\text{local-moment}}$  and  $\chi^{\text{dynamic}}$  indicating the predominance of the local moment.
- The  $t_m^+$  line indicates the end of the crossover between the bad insulator and the Mott phase as the growing interaction has less and less effect on the magnetic fluctuations period (reaching the atomic limit).

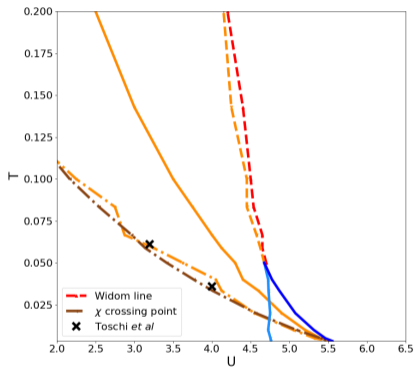


Figure 14 – Comparison the  $t_m^{max}$  line from our work with previous works

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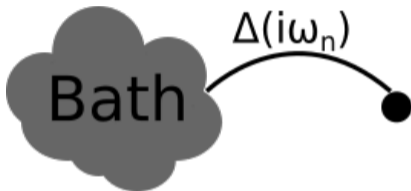


Figure 15 – Schematic representation of a site coupled to a bath

One can define the **timescale associated with the valence history** using the DMFT hybridization function  $\Delta(i\omega_n)$

$$t_{hyb} = -\frac{1}{\lim_{i\omega_n \rightarrow 0} \Im \Delta(i\omega_n)} \quad (6)$$

How do the two different timescales compare?

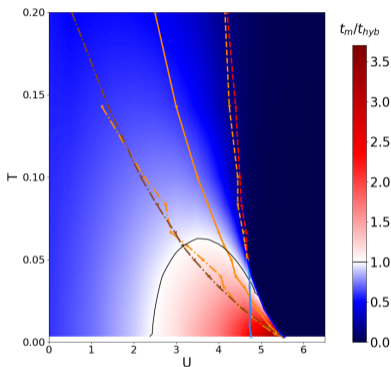


Figure 16 – Heatmap of the ratio between  $t_m$  and  $t_{hyb}$  in the DMFT phase diagram of the Hubbard model

- $t_m/t_{hyb} < 1$  : Magnetic fluctuations **faster** than valence fluctuations.
- $t_m/t_{hyb} > 1$  : Magnetic fluctuations **slower** than valence fluctuations.
- **Mott phase** : No valence fluctuations,  $t_m \ll t_{hyb}$
- **Low interaction and/or high temperature** :  $t_m < t_{hyb}$
- Appearance of a **dome** where  $t_{hyb} > t_m$  : valence fluctuations faster than magnetic fluctuations (adiabatic spin response).

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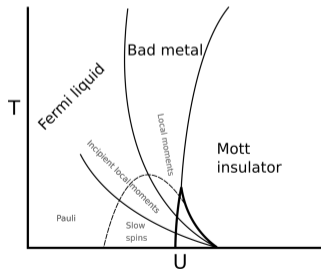
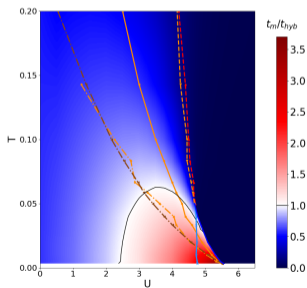
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- We provided a **new insight** into the phase diagram of the Hubbard **model** through the lens of **spin dynamics**.
- We identified a **new crossover line** within the Fermi liquid phase with a region with preponderant local moment
- We identified a regime in which the **spin dynamics is adiabatic**



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