Ab-initio description of monopole resonances in light- and medium-mass nuclei

Preliminary results

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Outline

- Introduction
- Formalism
- Preliminary results
- Conclusions

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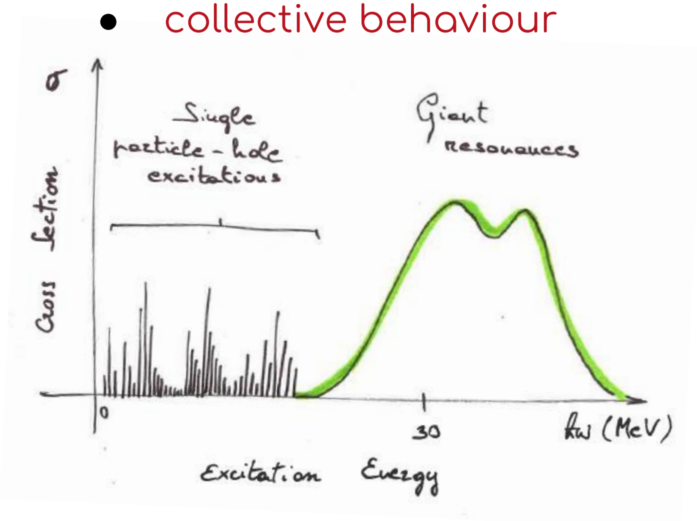
Introduction and Motivation

Dual nature of nucleus

- single-particle features
- collective behaviour



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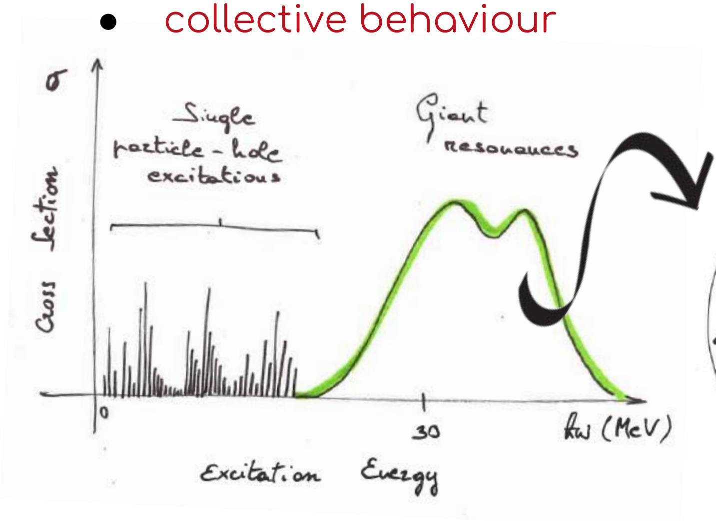
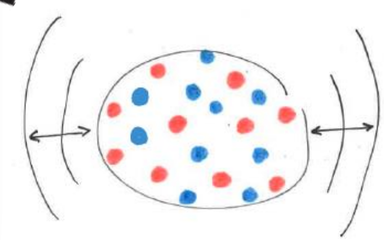


Giant Resonances (GRs)

clearest manifestation of collective motion

Liquid drop picture
vibrations, oscillations



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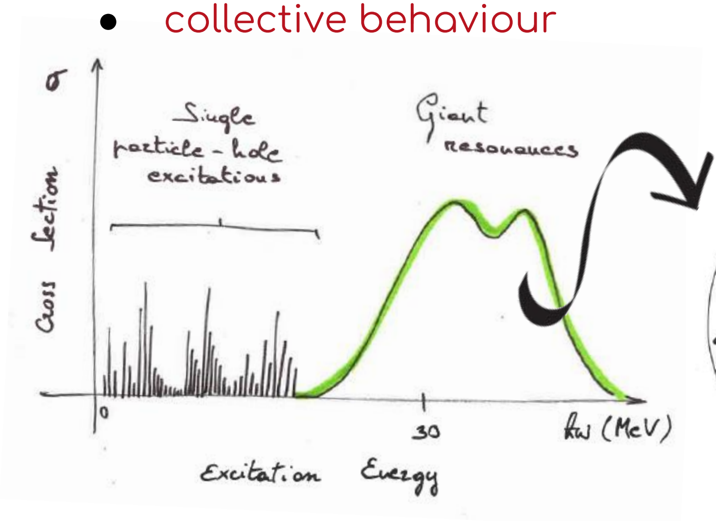

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decomposed in terms of angular momentum L

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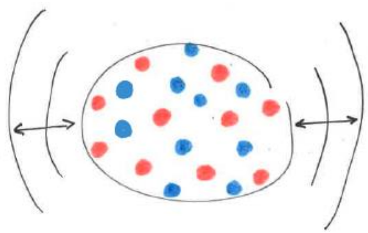

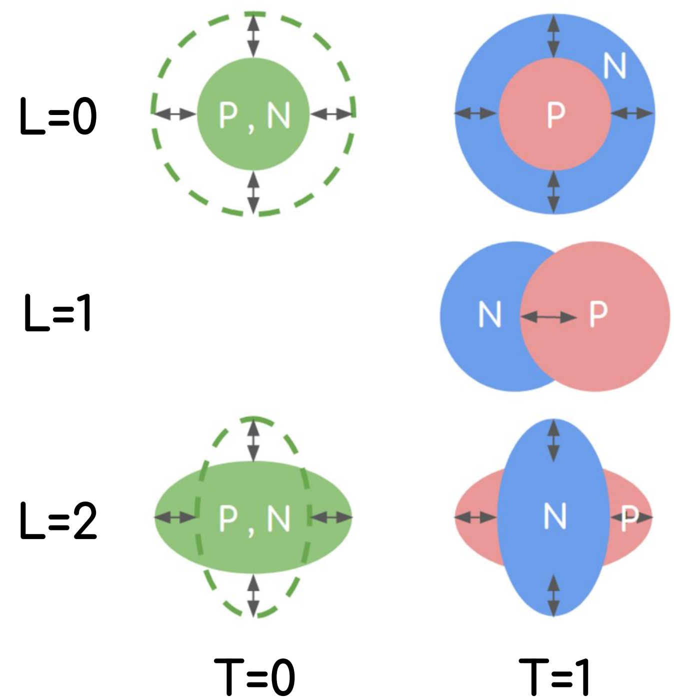

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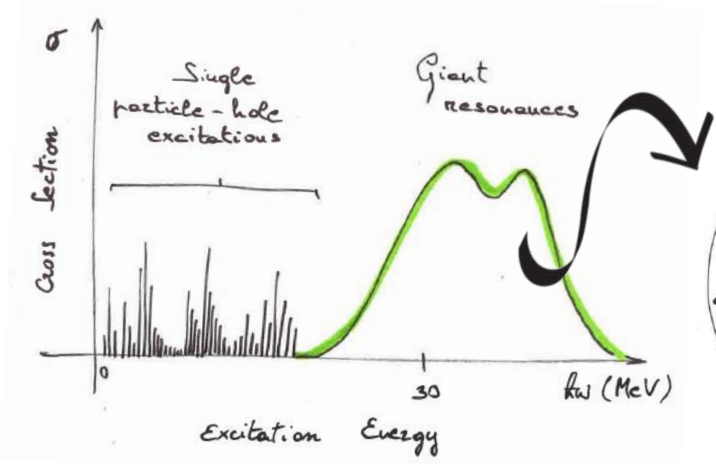

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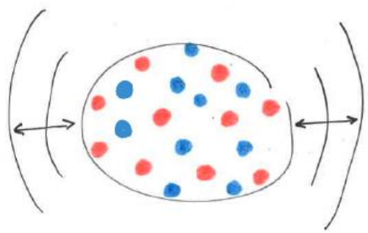
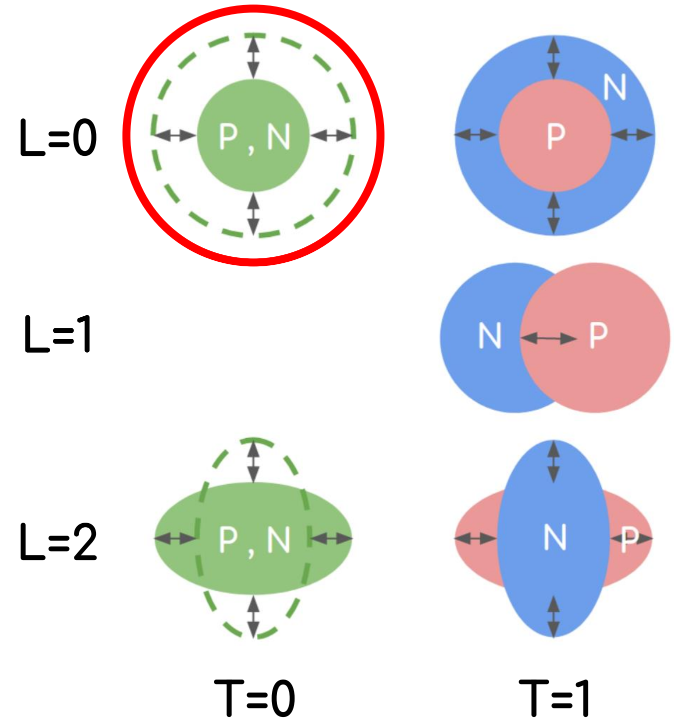
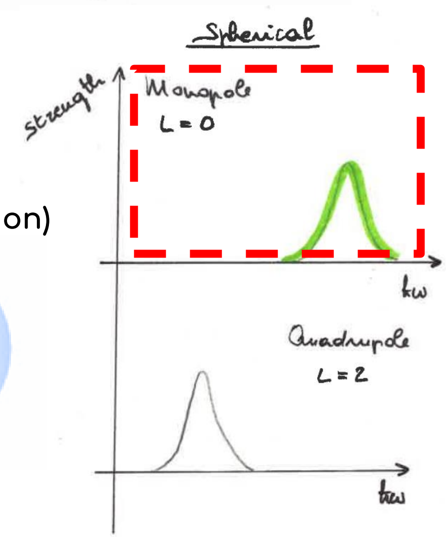




Spherical
(no deformation)



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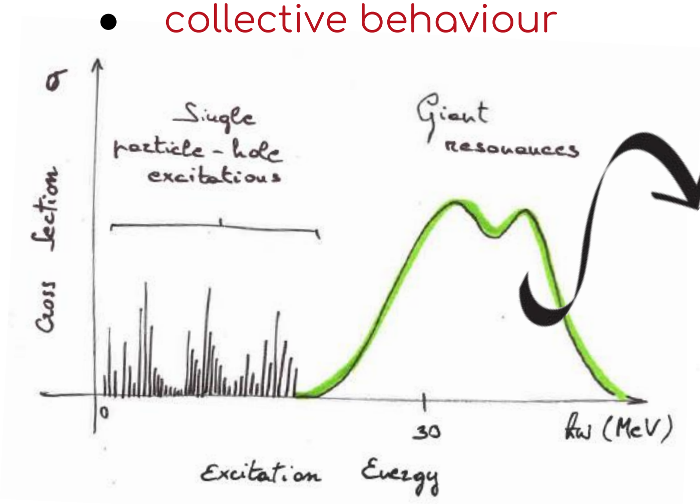

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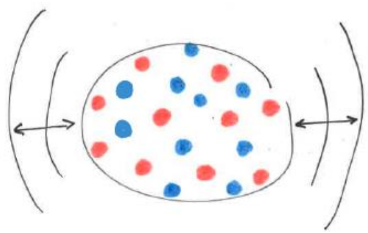

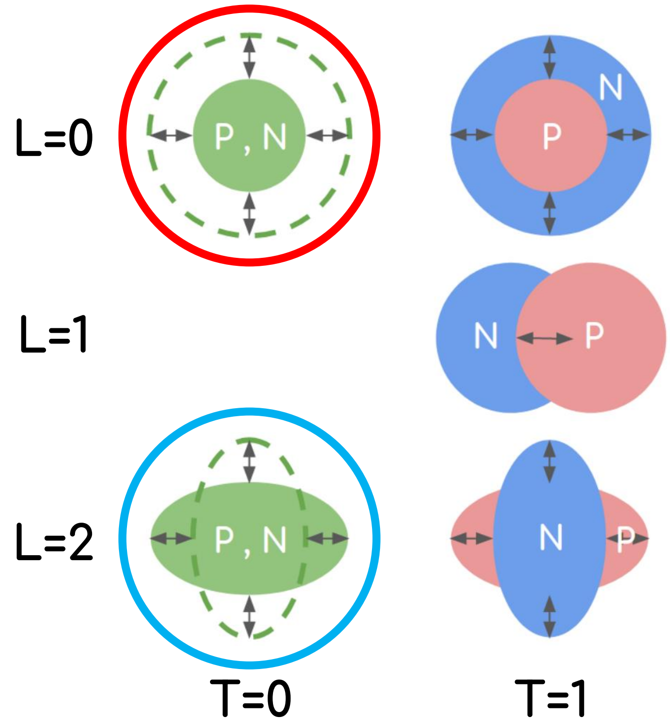

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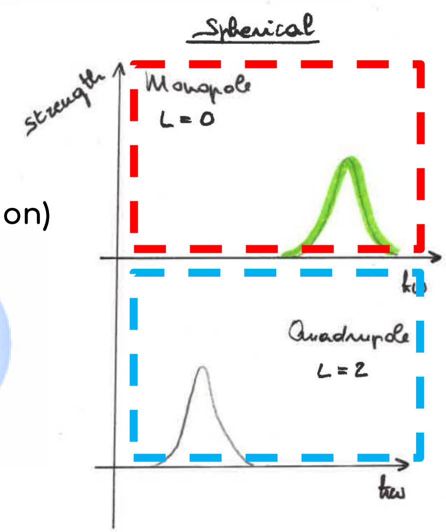

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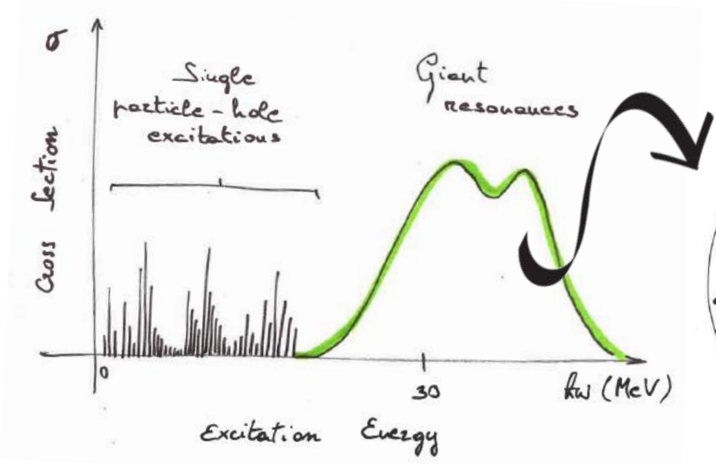

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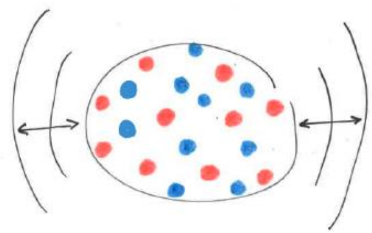

Deformation
↓
Coupling between different L

Spherical
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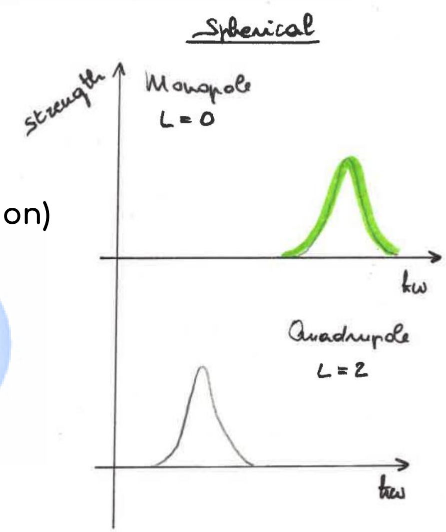
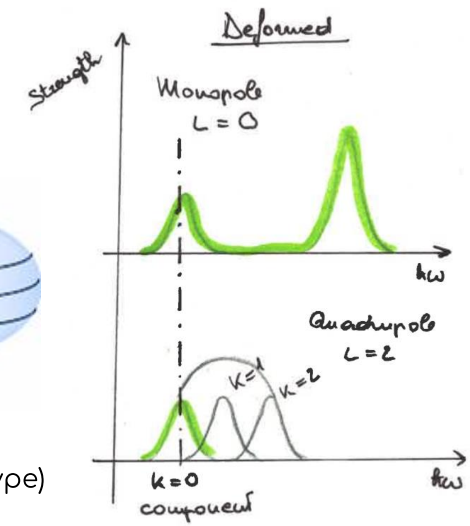


Spheroidal
(deformed)

Prolate
(cigar type)

Oblate
(pancake type)



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What is it ?

- Collective excitation (breathing mode)
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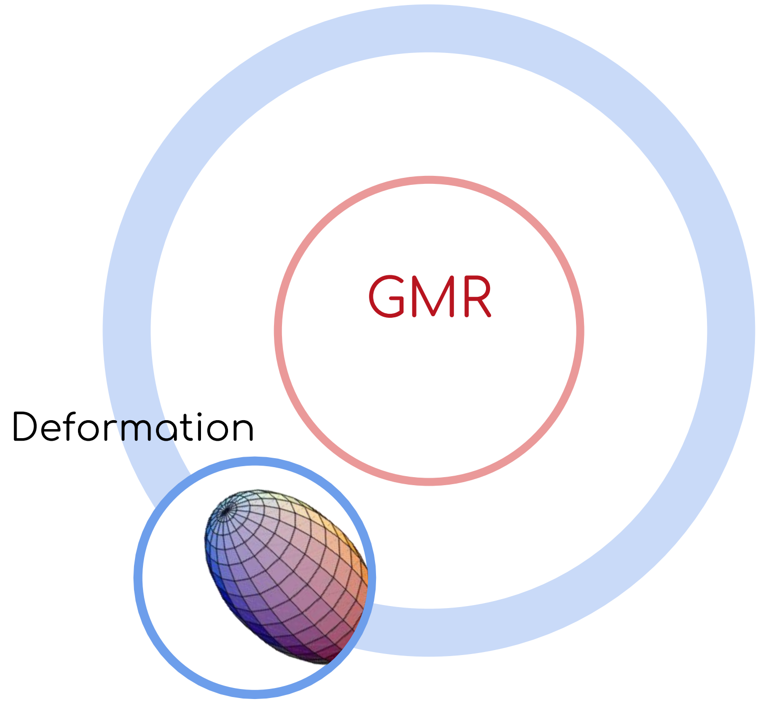

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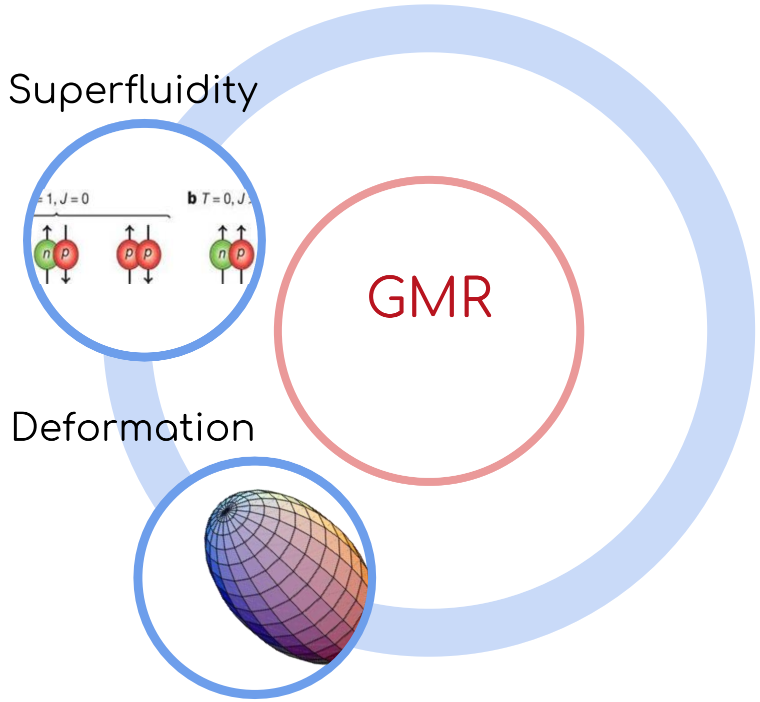

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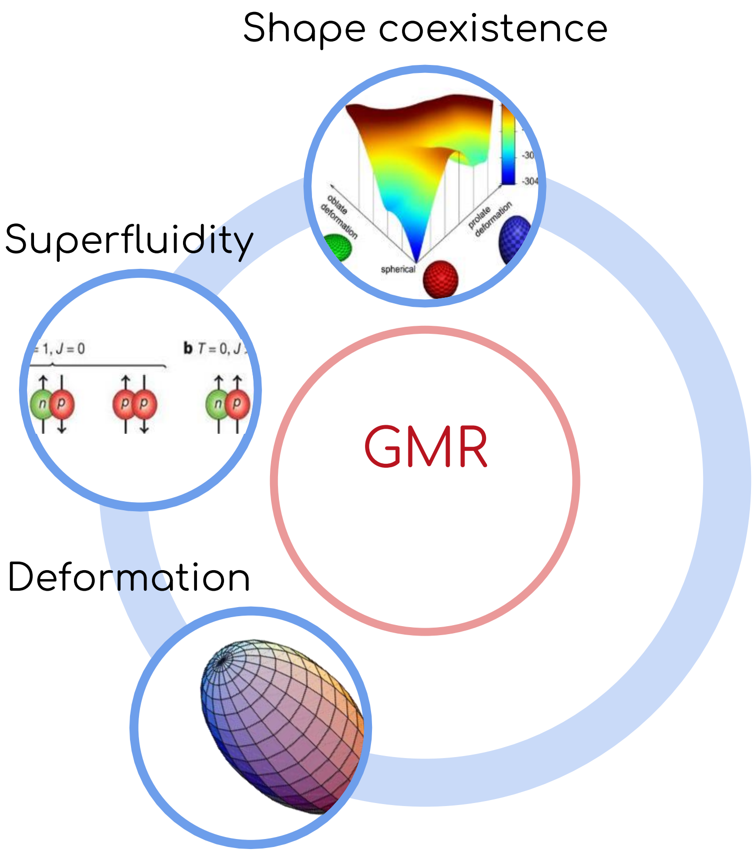

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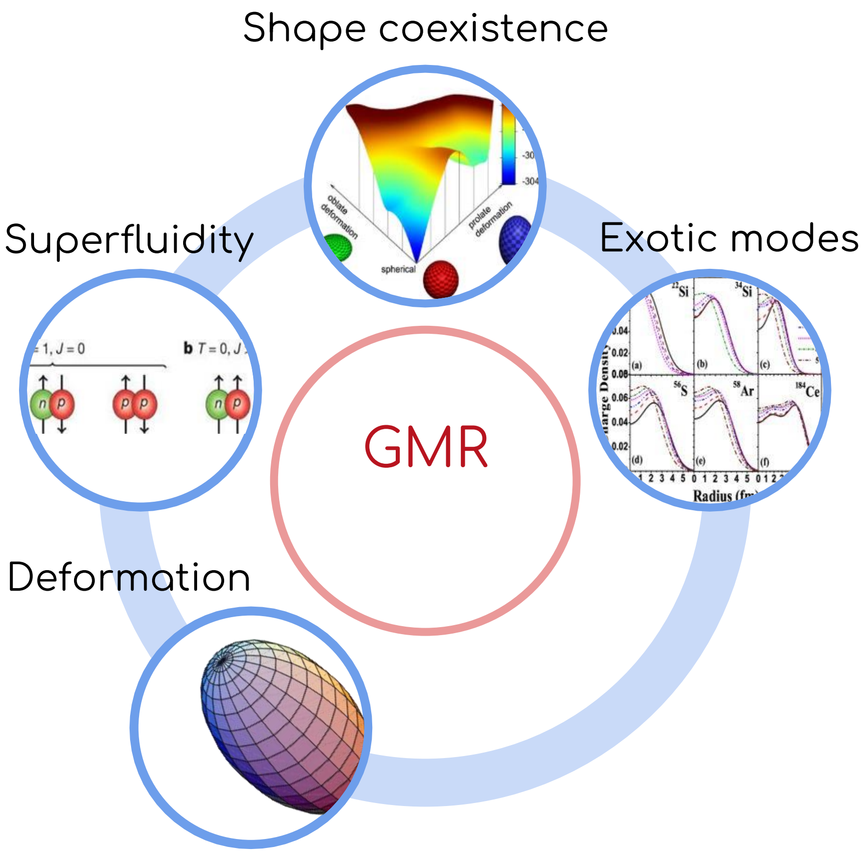

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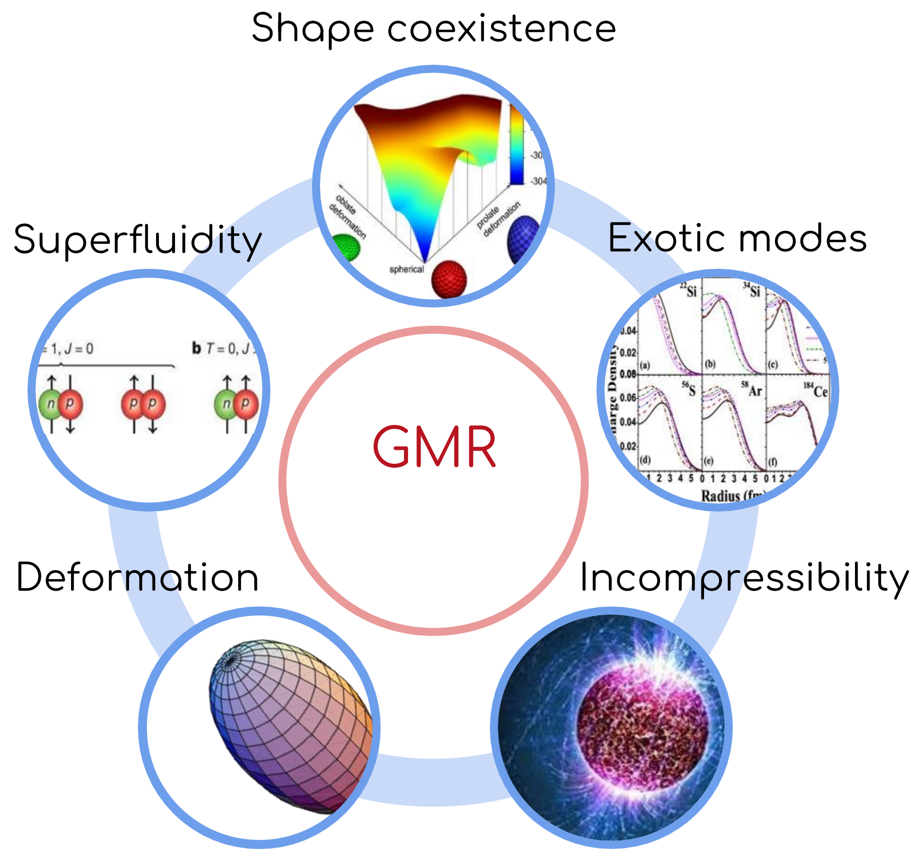

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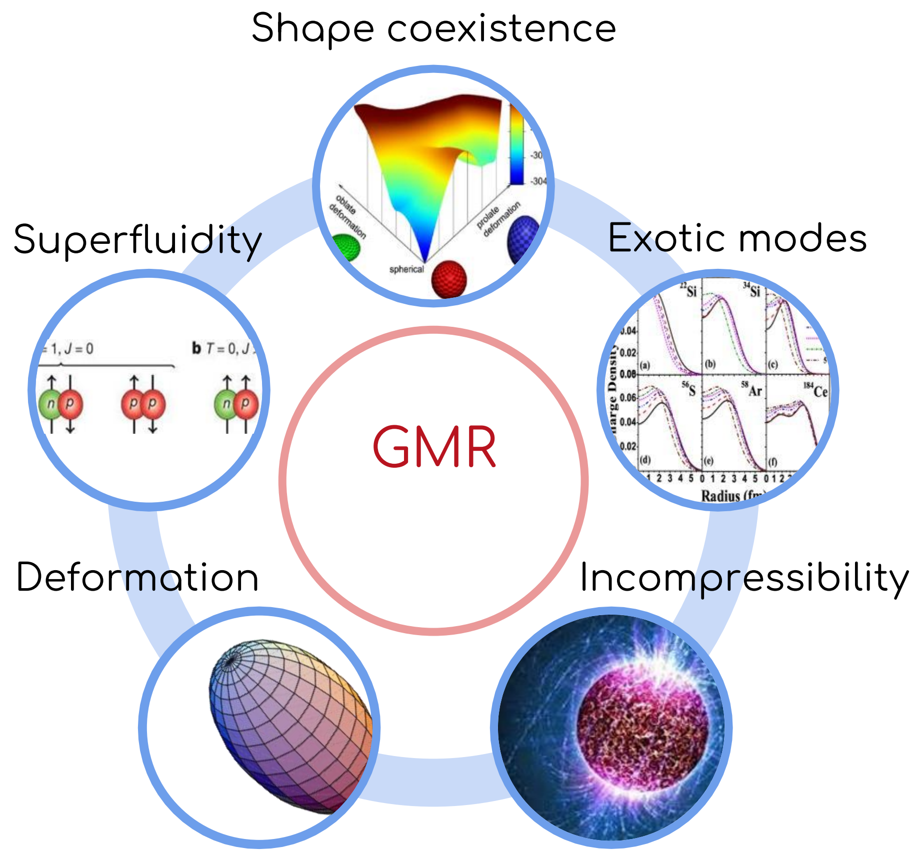

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- No systematic studies (EDF as well)
    - Very generic numerical codes needed
- **Ab-initio description still seminal**

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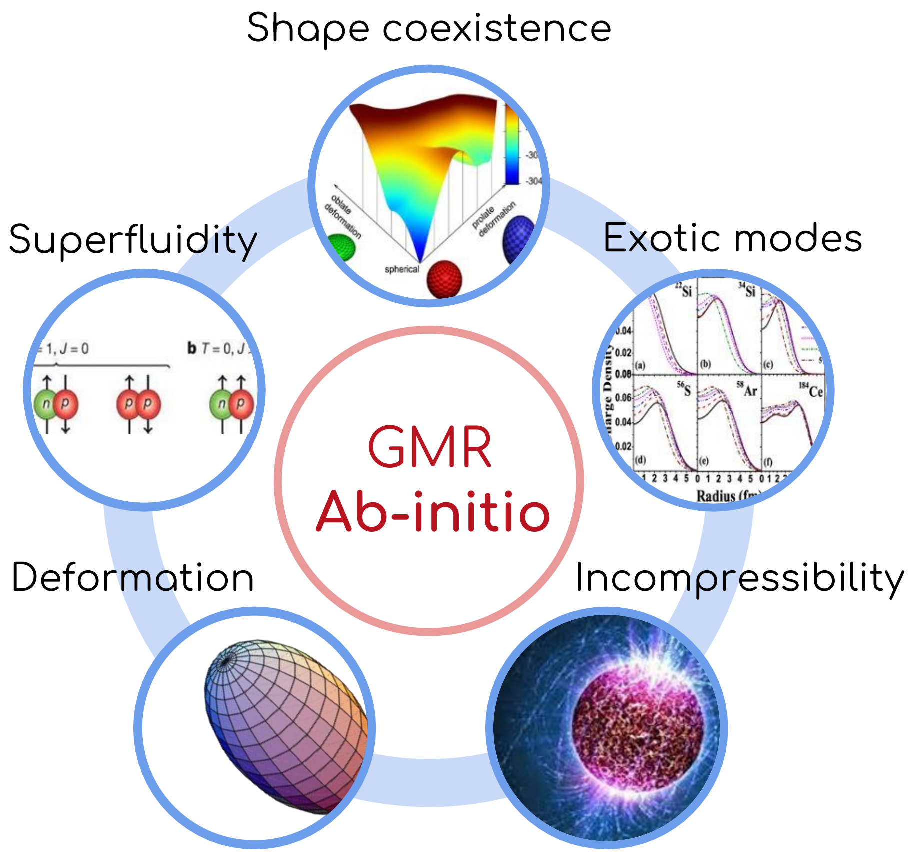

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Theoretical Frame

Ab-initio methods

Ab-initio methods have previously been introduced (see V. Somà's and P. Demol's talks)

$$H|\Psi_\nu\rangle = E_\nu |\Psi_\nu\rangle$$

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GMR has historically been studied within **EDF** theory [Garg, Colò, 2018]

Ab-initio unicum: (Q)RPA for spherical systems

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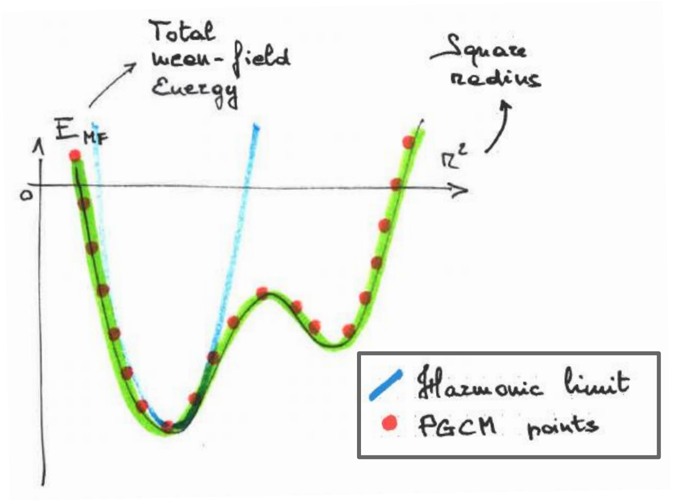
Present goal: First systematic ab-initio study of the GMR

- PGCM — Projected GCM, superfluid version of NOCI
- QRPA — Superfluid version of RPA

PGCM vs QRPA

Schrödinger equation $H\left|\Psi_\nu\right\rangle = E_\nu \left|\Psi_\nu\right\rangle$



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PGCM

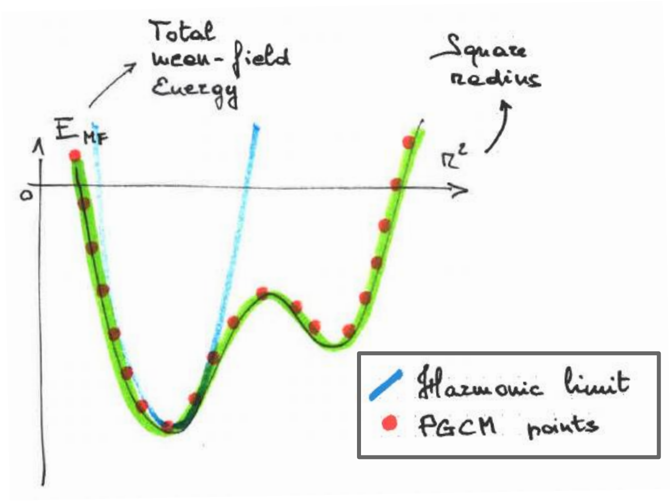
$$|\Psi_\nu\rangle \equiv \sum_{r^2,q} f_\nu(r^2,q)\,|\Phi(r^2,q)\rangle$$

r^2 to study GMR

q to couple to other modes

Symmetry breaking and restoration

Variational method



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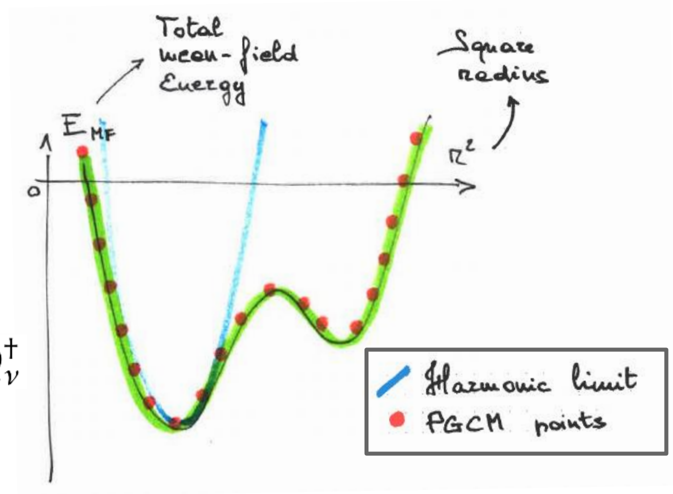

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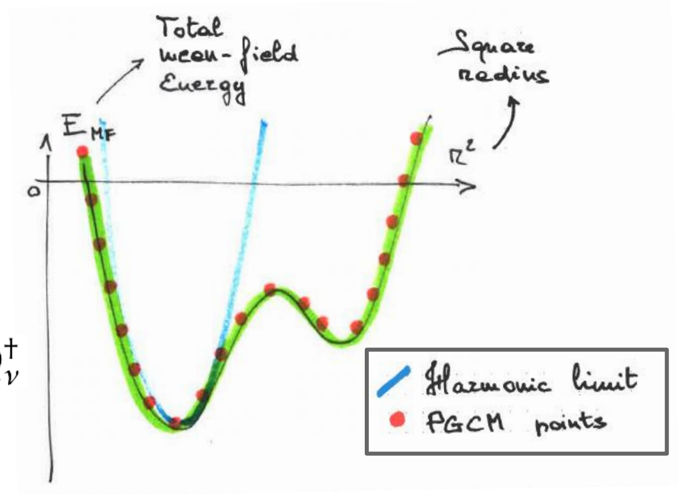

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Pros and Cons

PGCM	QRPA
Handle anharmonicities and shape coexistence	Harmonic limit of GCM [Brink, Weiguny, 1968]
Select on few collective coordinates	All coordinates are explored
Symmetries are restored	Symmetries are not restored
Computationally expensive	Low computational cost

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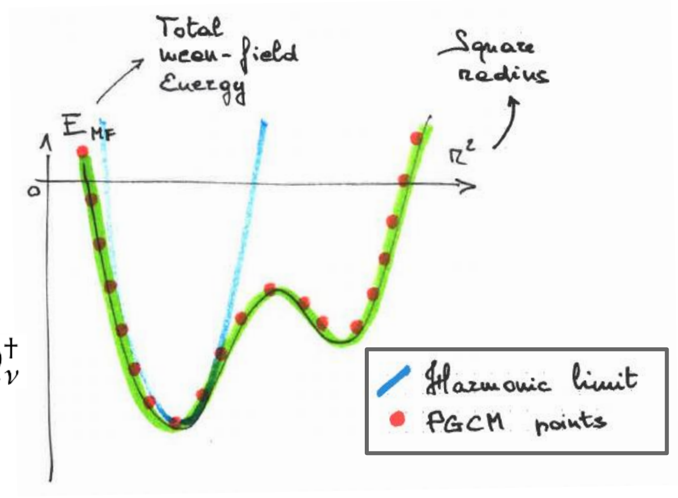

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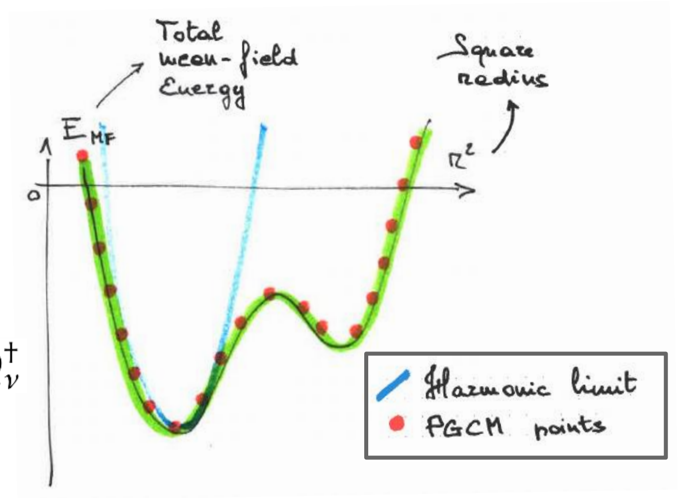

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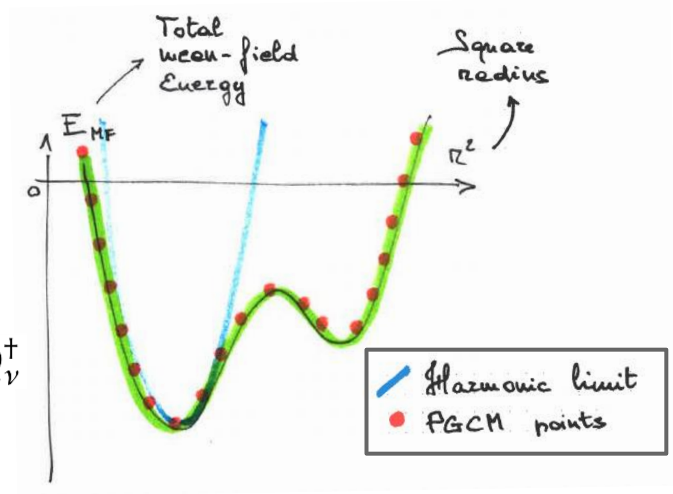

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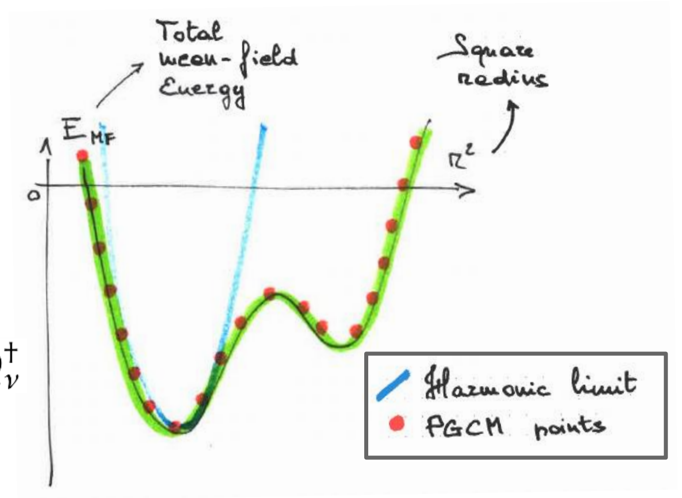

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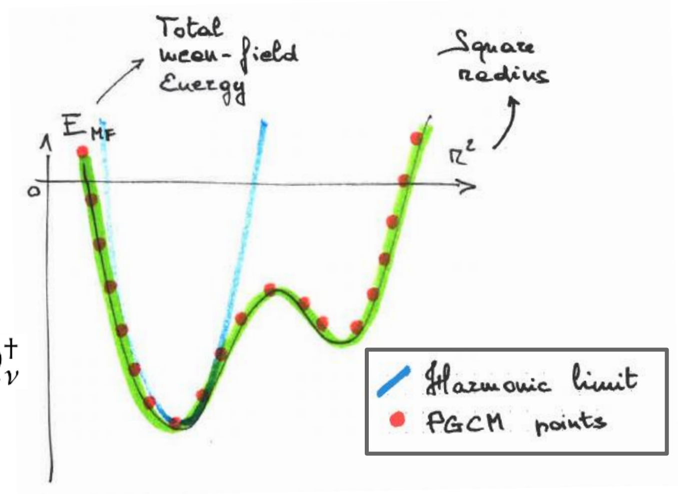

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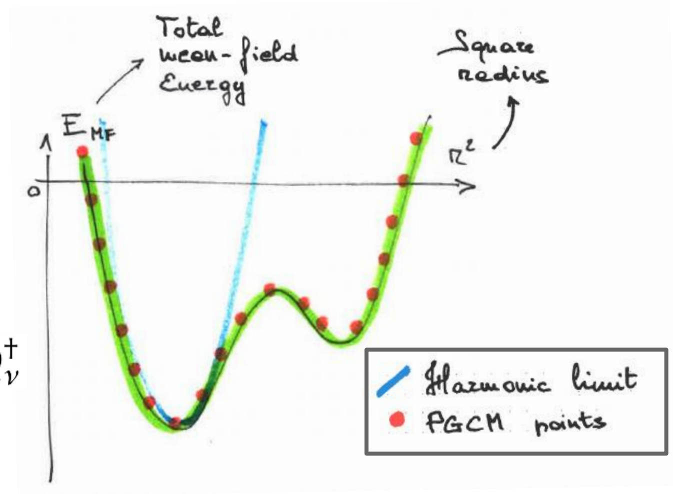

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General implementation, can access

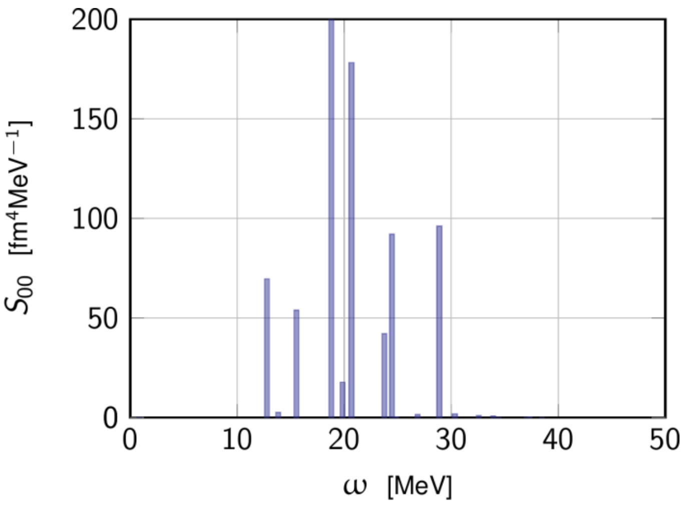
1. Doubly-closed-shell nuclei
2. Singly-open-shell nuclei
3. Doubly-open-shell nuclei

Moments and Strength

- Studied quantity: **monopole strength**

$$S_{00}(\omega) \equiv \sum_{\nu} |\langle \Psi_\nu | r^2 | \Psi_0 \rangle|^2 \delta(E_\nu - E_0 - \omega)$$

  - Transition amplitudes: height of peaks
  - Energy difference: position of peaks



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JM=00

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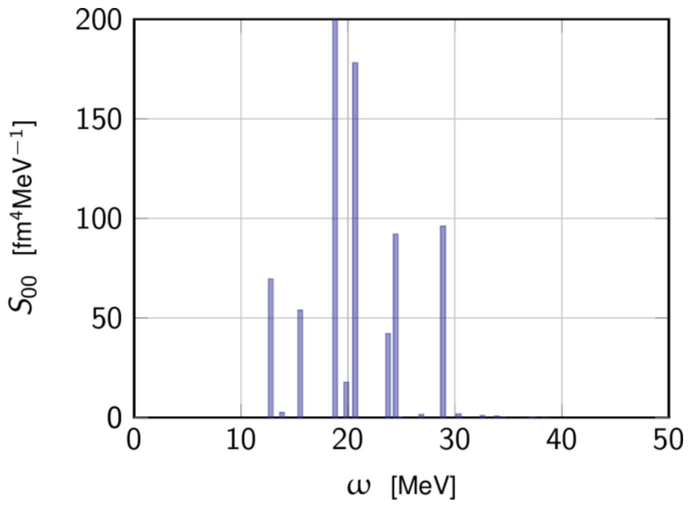

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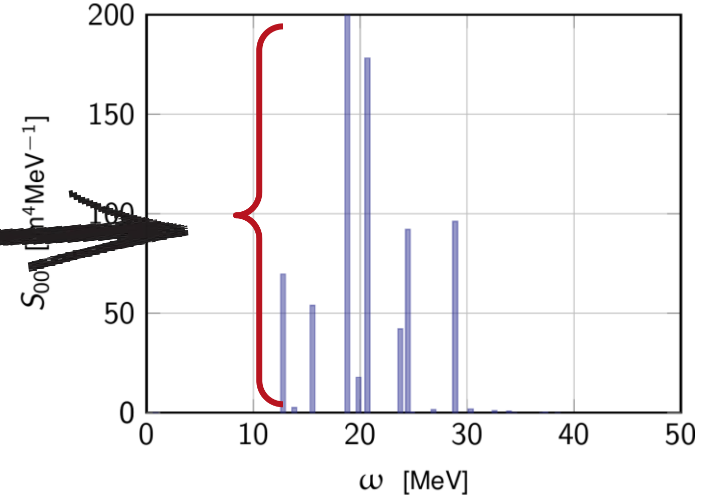

Moments and Strength

- Studied quantity: **monopole strength**

$$S_{00}(\omega) \equiv \sum_{\nu} |\langle \Psi_\nu | r^2 | \Psi_0 \rangle|^2 \delta(E_\nu - E_0 - \omega)$$

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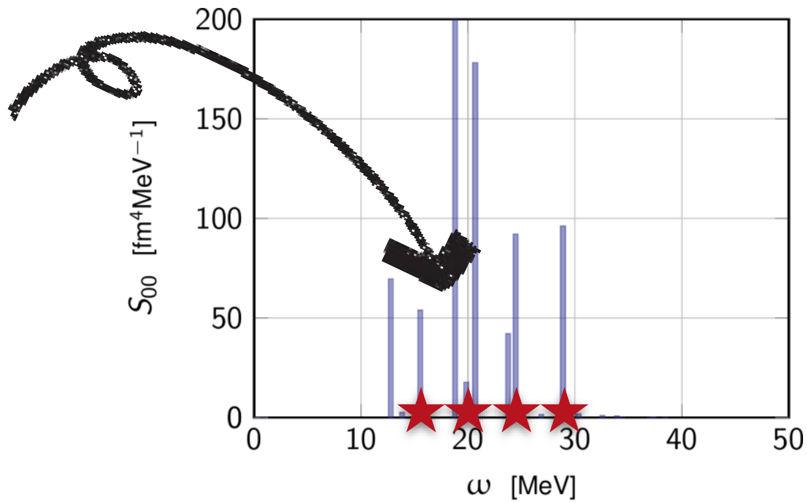

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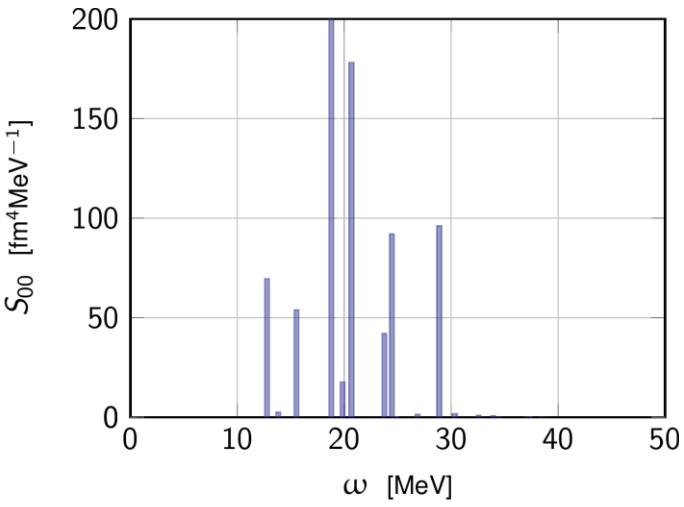

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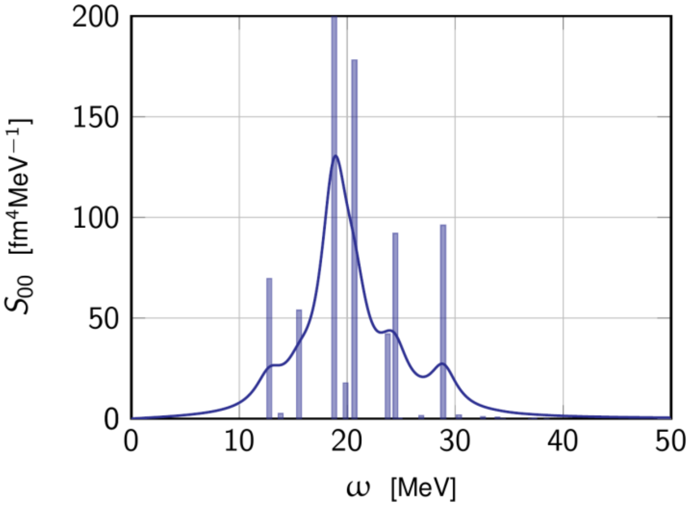

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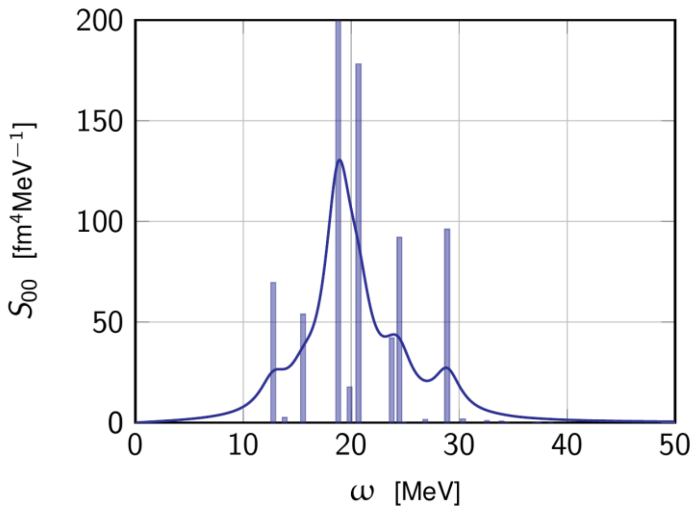
- Related moments

$$\begin{aligned} m_k &\equiv \int_0^\infty S_{00}(\omega)\, \omega^k \, d\omega \\ &= \sum_\nu (E_\nu - E_0)^k |\langle \Psi_\nu | r^2 | \Psi_0 \rangle|^2 \\ &\equiv \langle \Psi_0 | \check{M}_k(i,j) | \Psi_0 \rangle \end{aligned}$$



[Bohigas et al., 1979]

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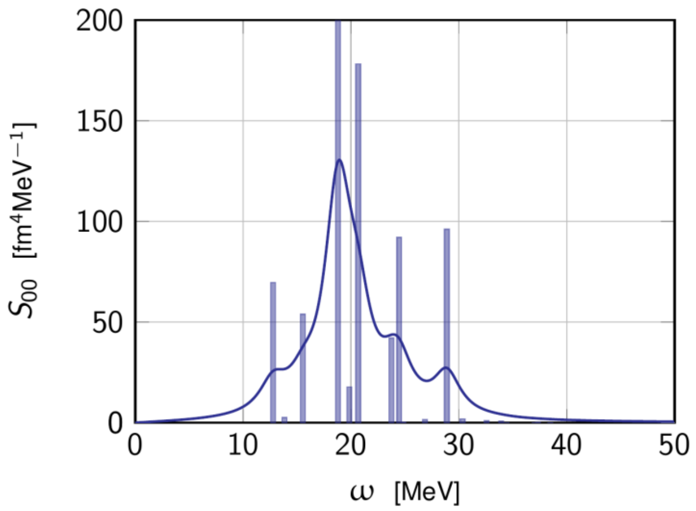

- Related moments $m_k \equiv \int_0^\infty S_{00}(\omega)\,\omega^k\,d\omega$

$$= \sum_\nu (E_\nu - E_0)^k |\langle\Psi_\nu|r^2|\Psi_0\rangle|^2 \quad \longrightarrow \text{Must know excited states}$$

$$\equiv \langle\Psi_0|\breve{M}_k(i,j)|\Psi_0\rangle$$

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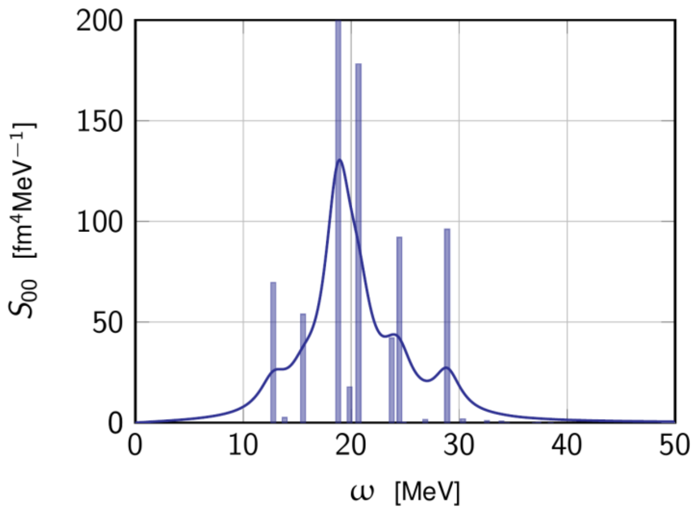

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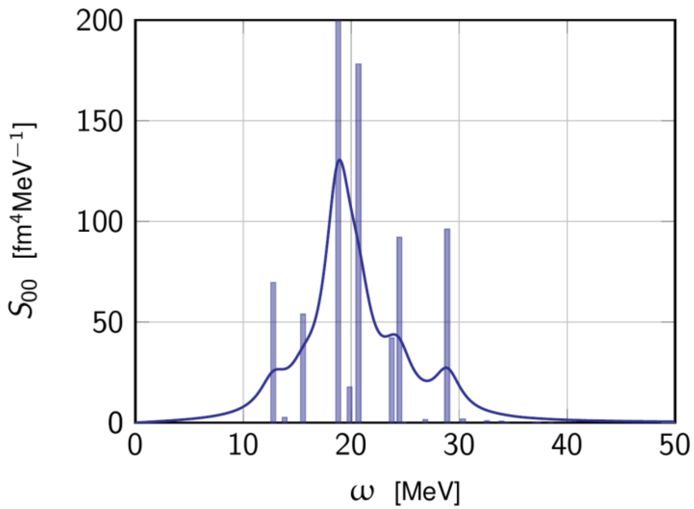

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$$\breve{M}_k(i,j) \equiv (-1)^i C_i C_j \qquad \forall \; k \geq 0$$

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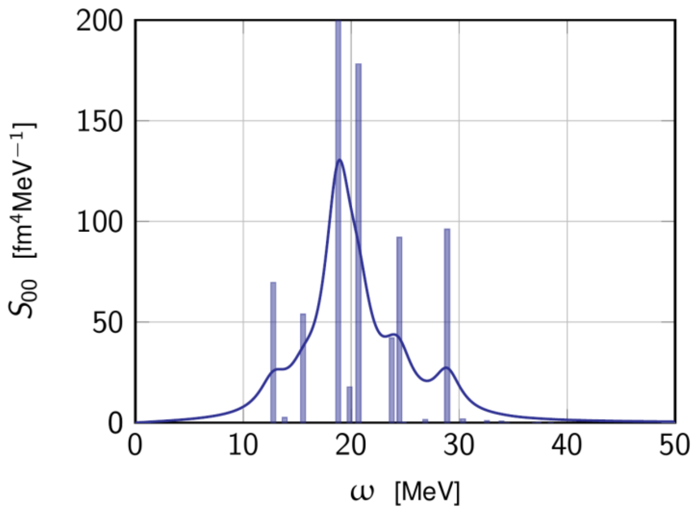

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Encode the main physical features of the strength

$$\bar{E}_1 = \frac{m_1}{m_0} \qquad \sigma^2 = \frac{m_2}{m_0} - \left(\frac{m_1}{m_0}\right)^2 \geq 0$$

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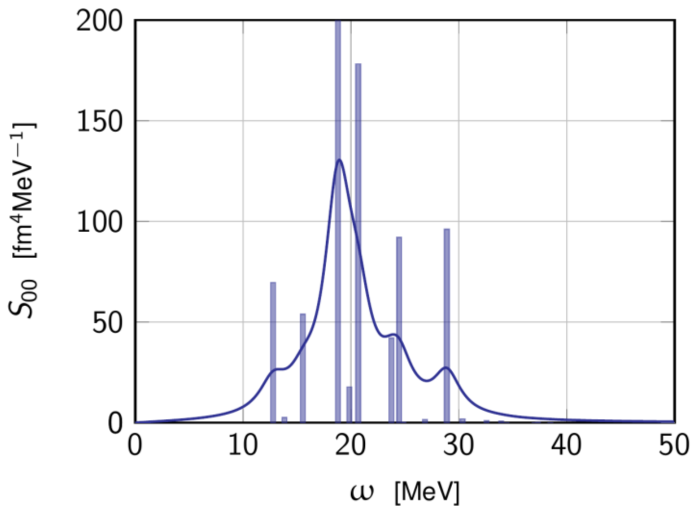

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First comparison ever of the two approaches !

Derived and **implemented** in an ab-initio PGCM code

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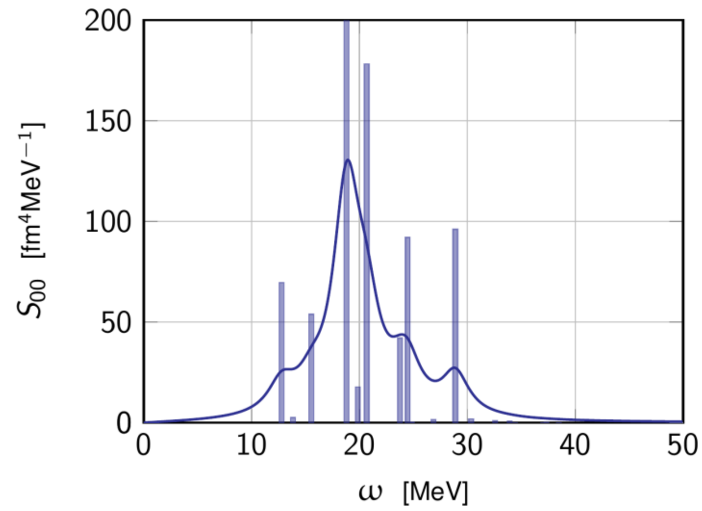

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Not discussed in the present talk

First comparison ever of the two approaches !

Derived and **implemented** in an ab-initio PGCM code

Outline

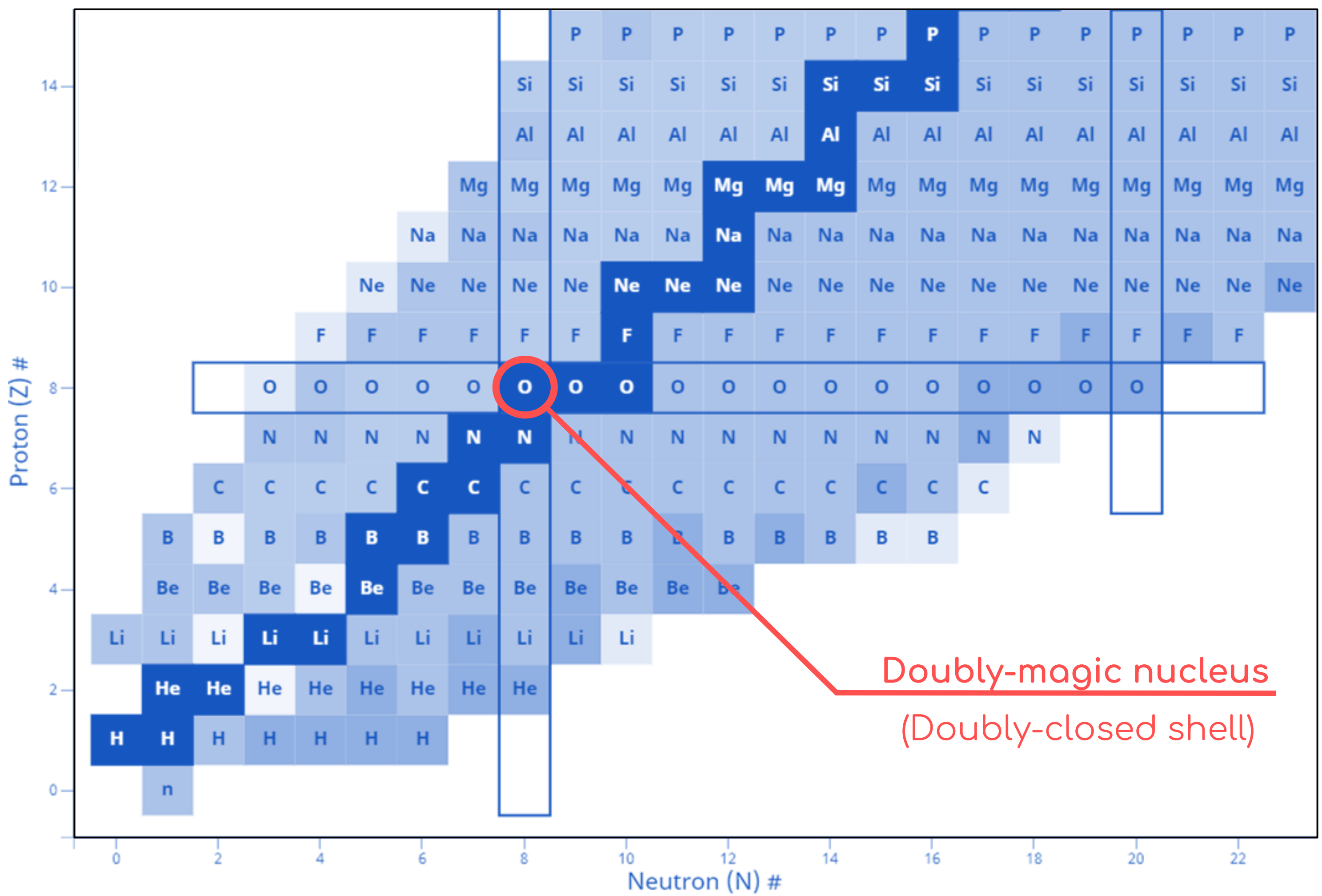
- Introduction
- Formalism
- **Preliminary results**
- Conclusions

Common features

PGCM and QFAM have **consistent numerical settings**

- One-body spherical harmonic oscillator basis
  - $e_{max} = 10$
  - $\hbar\omega = 20$ MeV
- Chiral **two-plus-three**-nucleon **in-medium** interaction
  - T. Hüther, K. Vobig, K. Hebeler, R. Machleidt and R. Roth, "Family of chiral two-plus three-nucleon interactions for accurate nuclear structure studies", *Phys. Lett. B*, 808, 2020
  - M. Frosini, T. Duguet, B. Bally, Y. Beaujeault-Taudière, J.-P. Ebran and V. Somà, "In-medium k-body reduction of n-body operators", *The European Physical Journal A*, *57*(4), 2021
- Only monopole strength is addressed
- The PGCM wavefunction explores the β_2 and r^2 **collective** coordinates (quadrupolar coupling)

Benchmarking ^{16}O

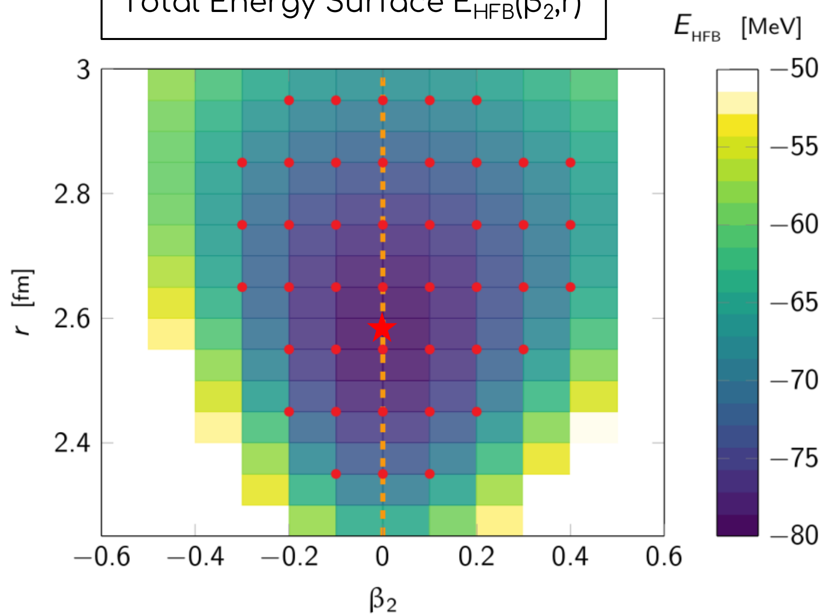


Benchmarking ^{16}O

Difficulty

☐ Benchmark on existing spherical QRPA code

Total Energy Surface $E_{HFB}(\beta_2,r)$



Results

- Single spherical harmonic energy minimum

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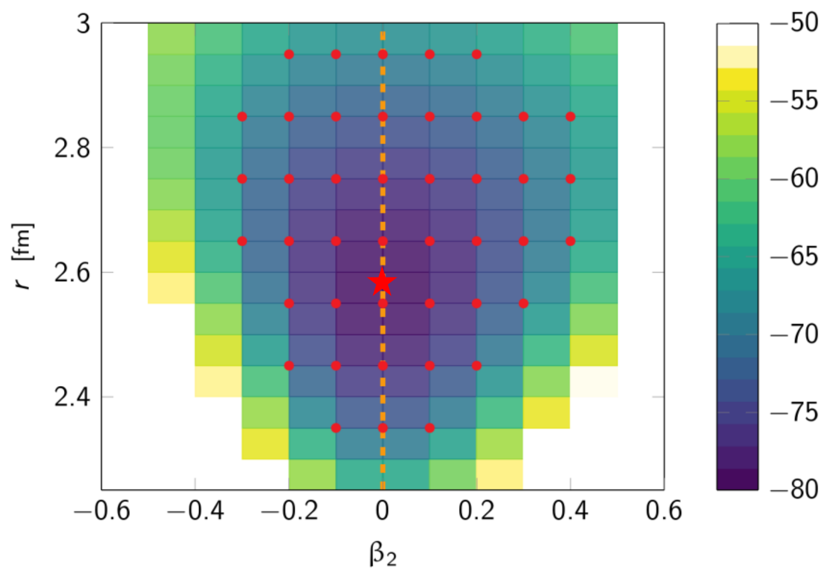
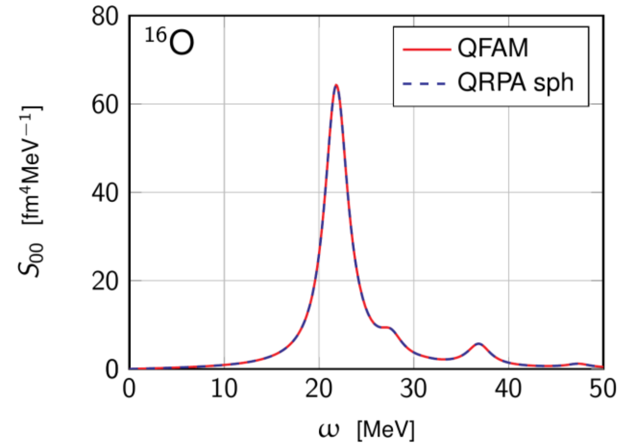


Monopole Strength



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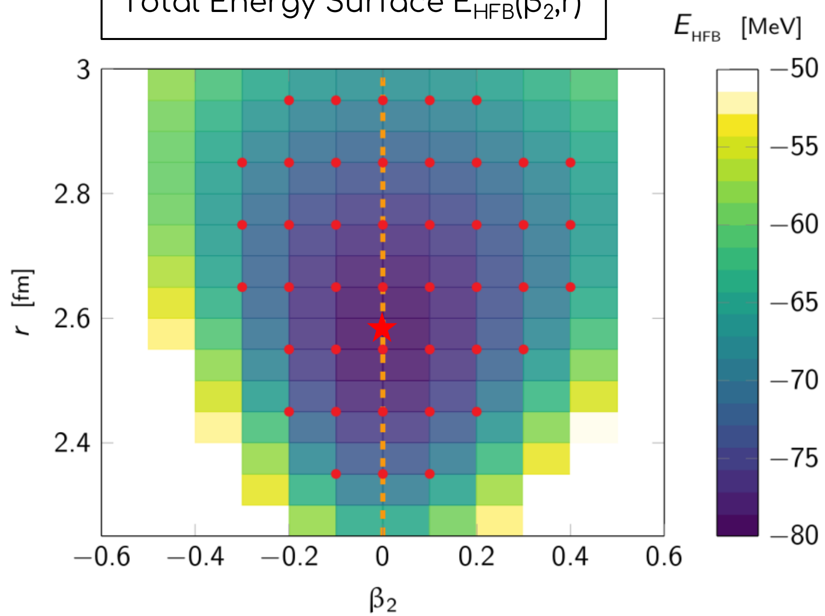

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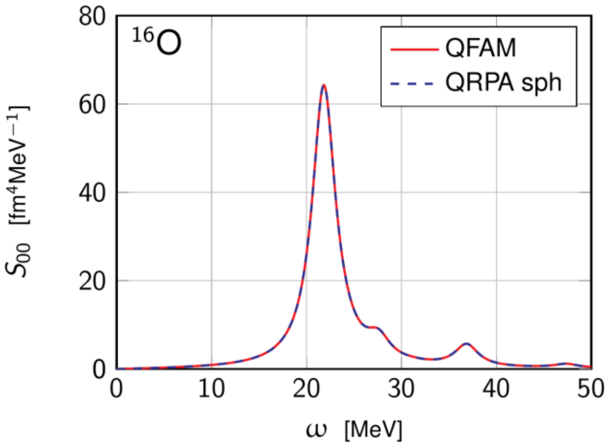

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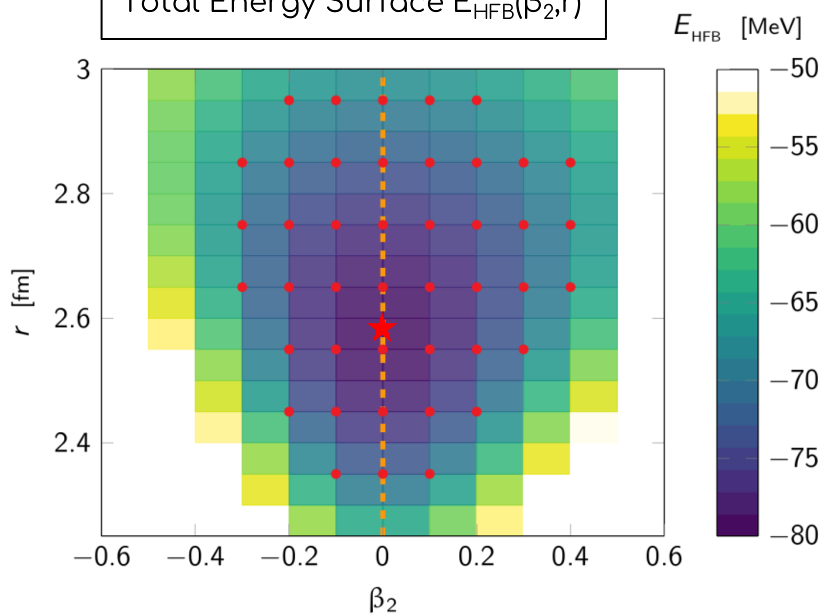

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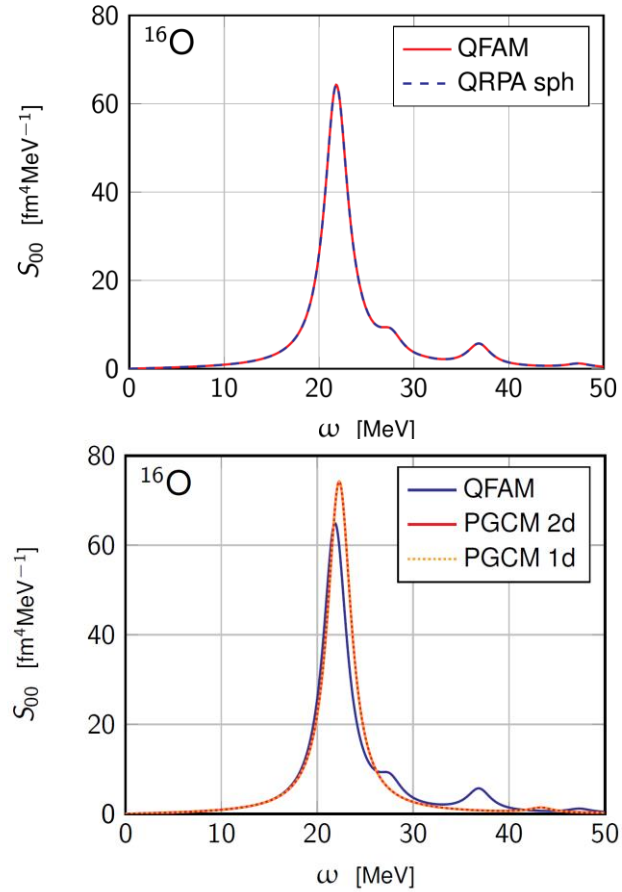

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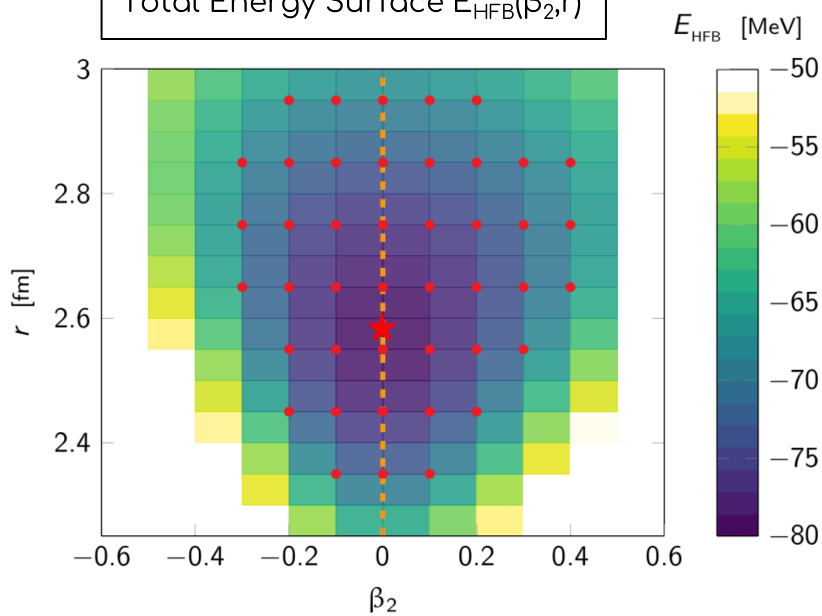

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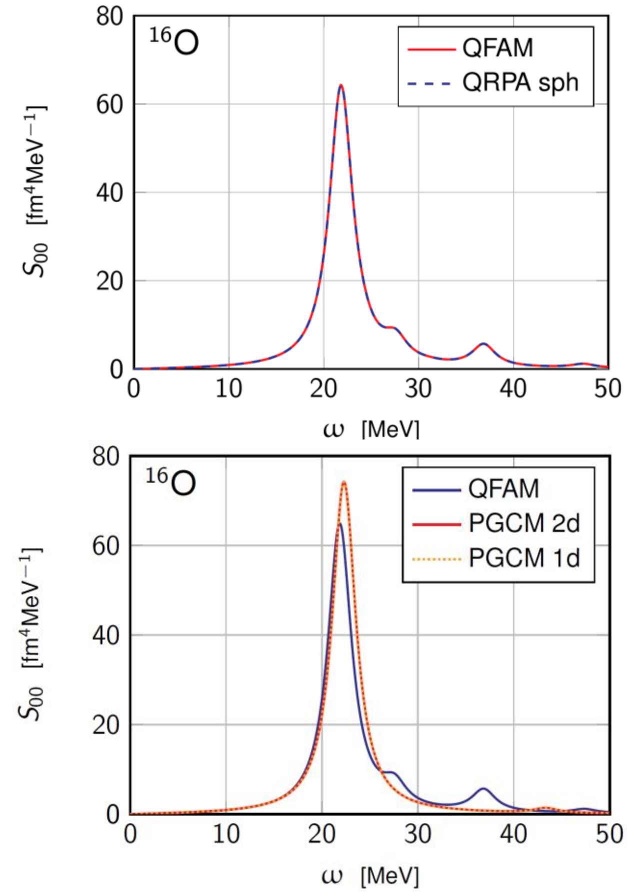

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Benchmarking ^{16}O

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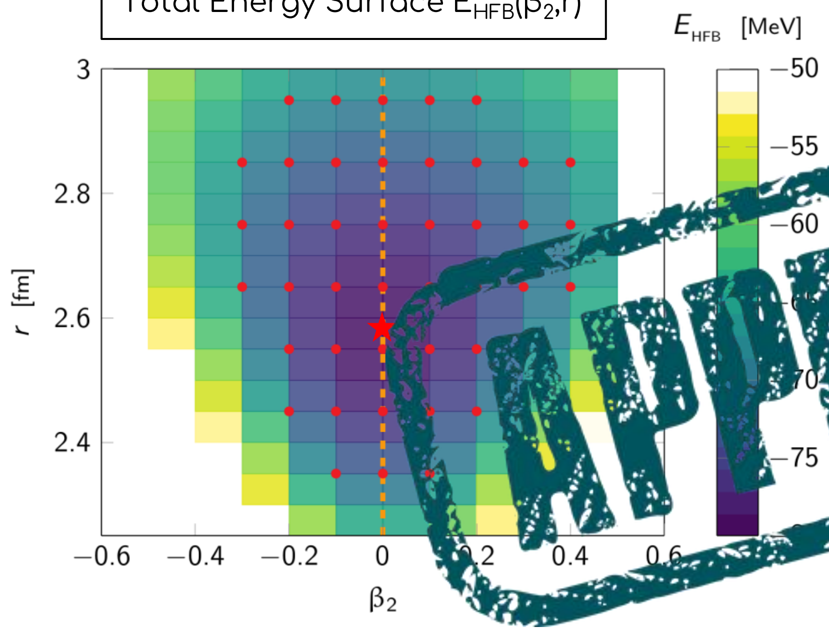

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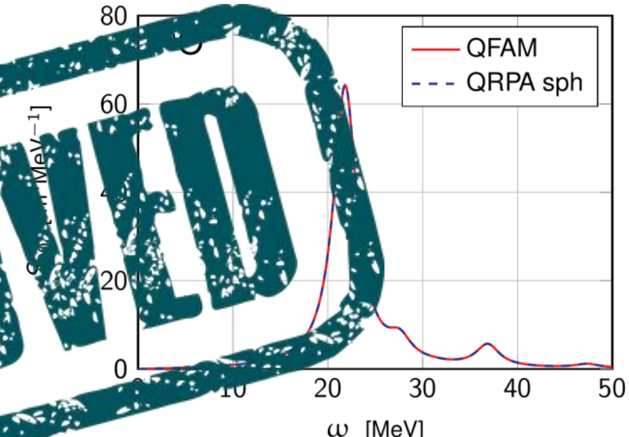

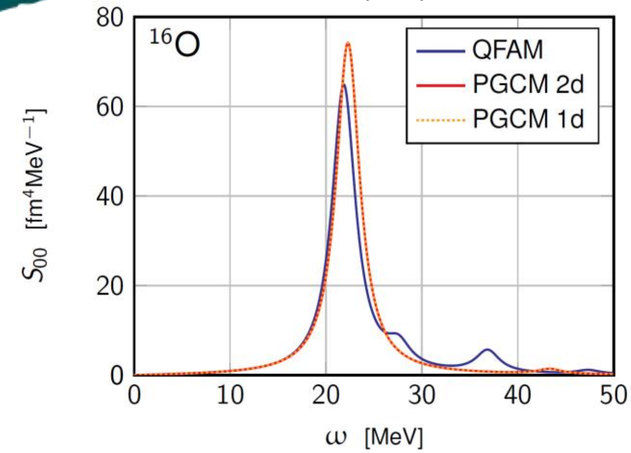

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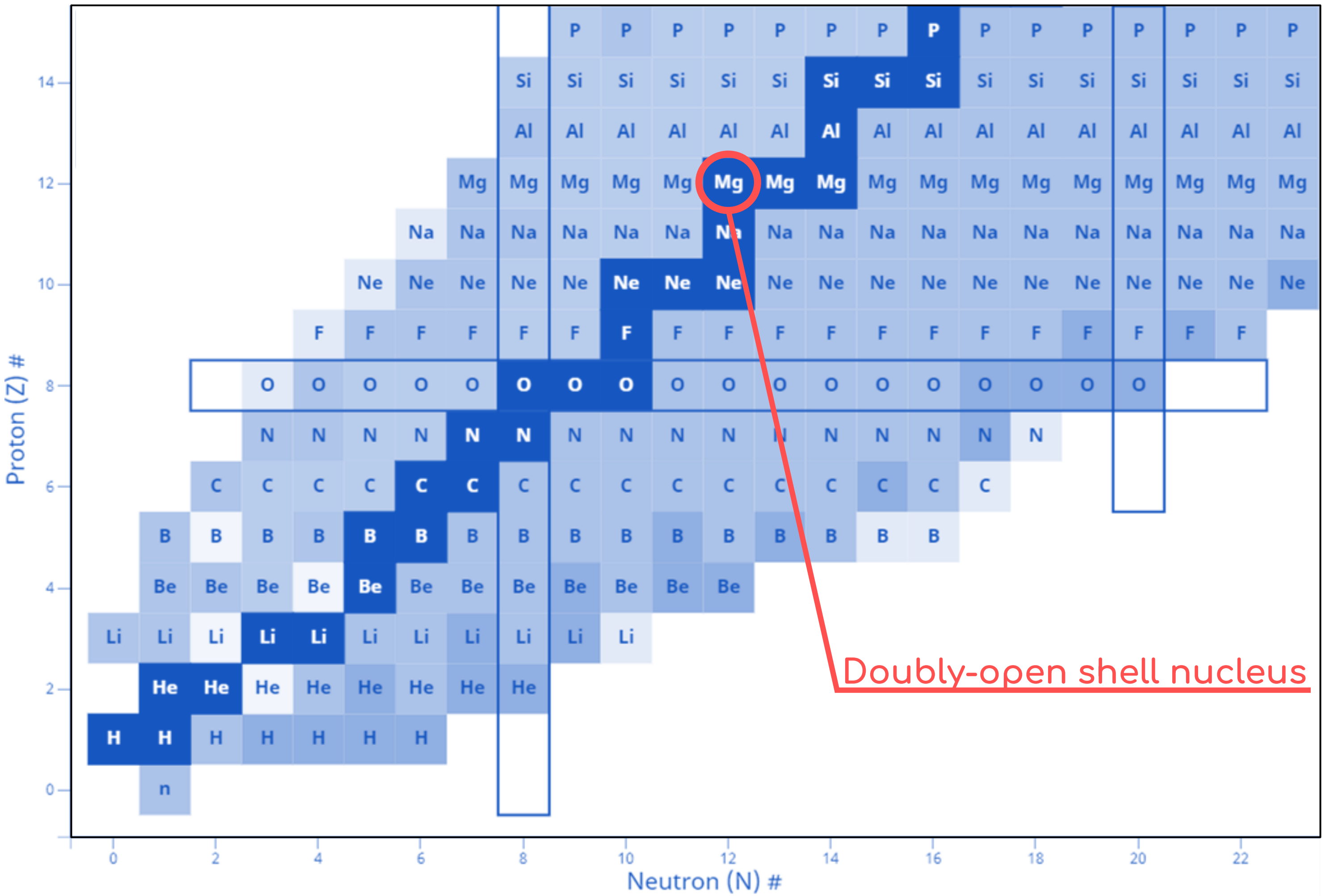
Deformation effects in ^{24}Mg



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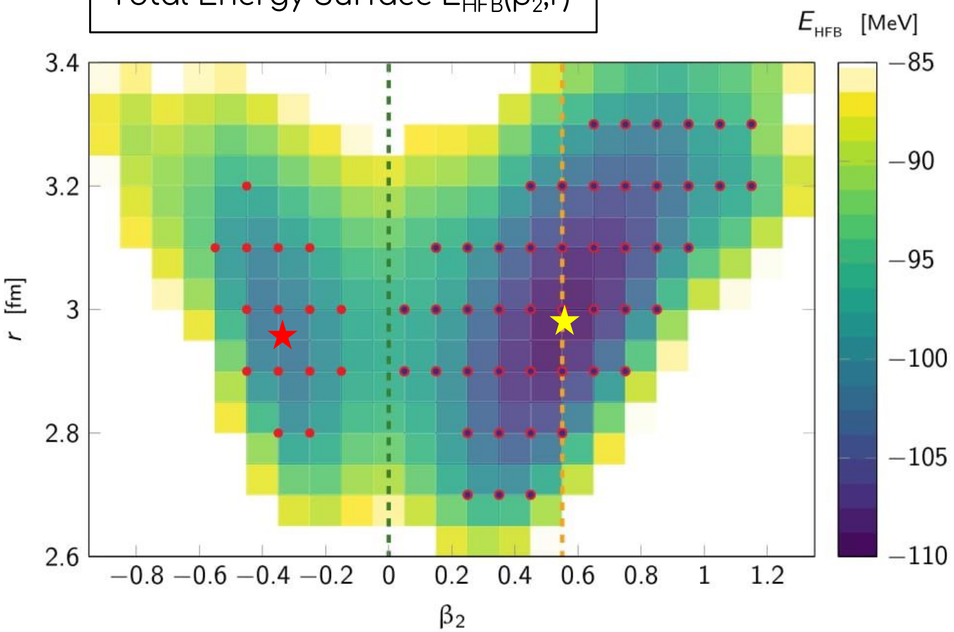

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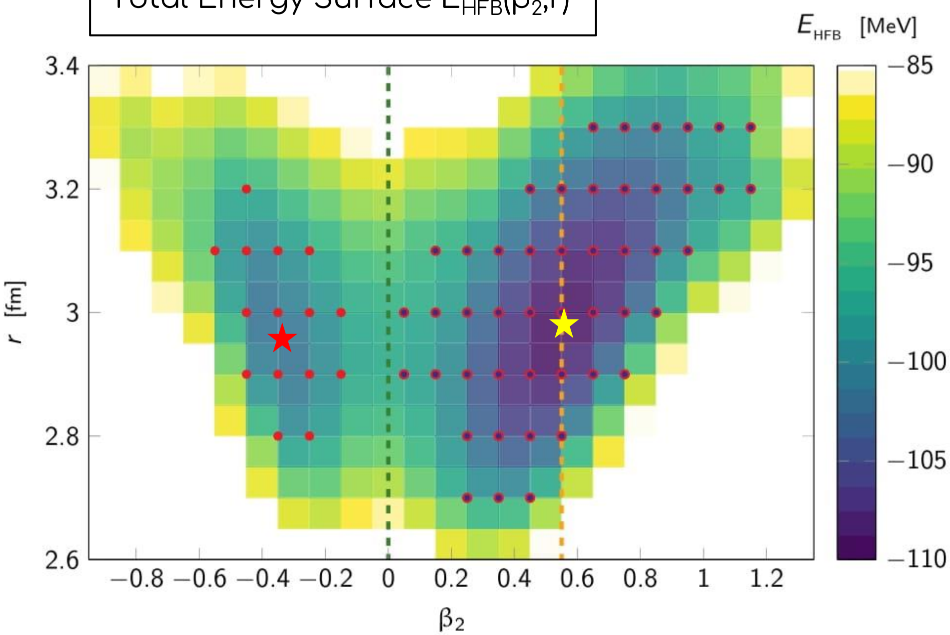

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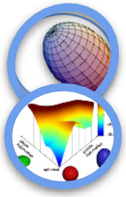

Deformation

Shape coexistence ? (1)

(1) [Dowie et al., 2020]

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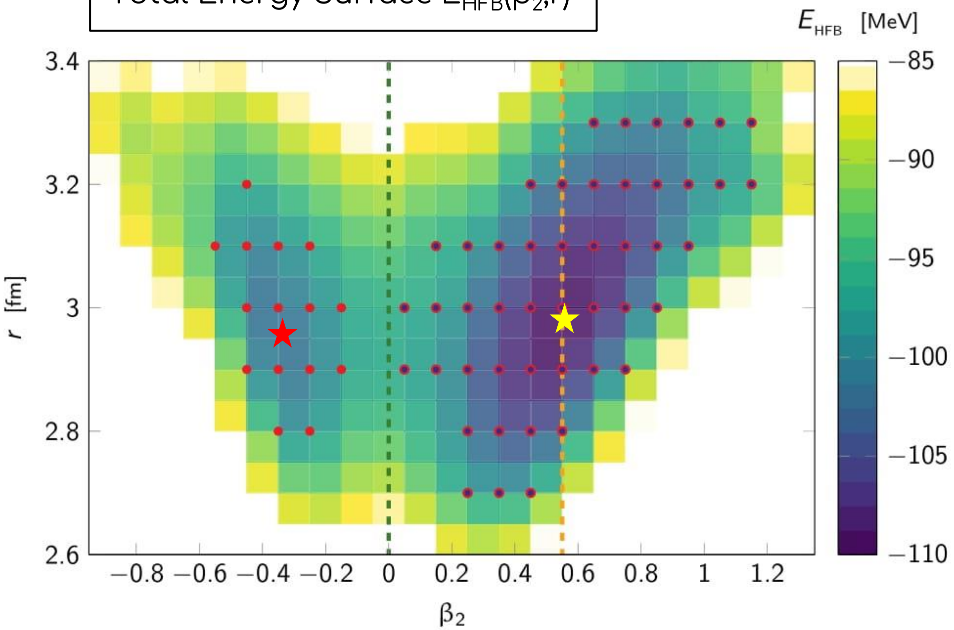

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Difficulty

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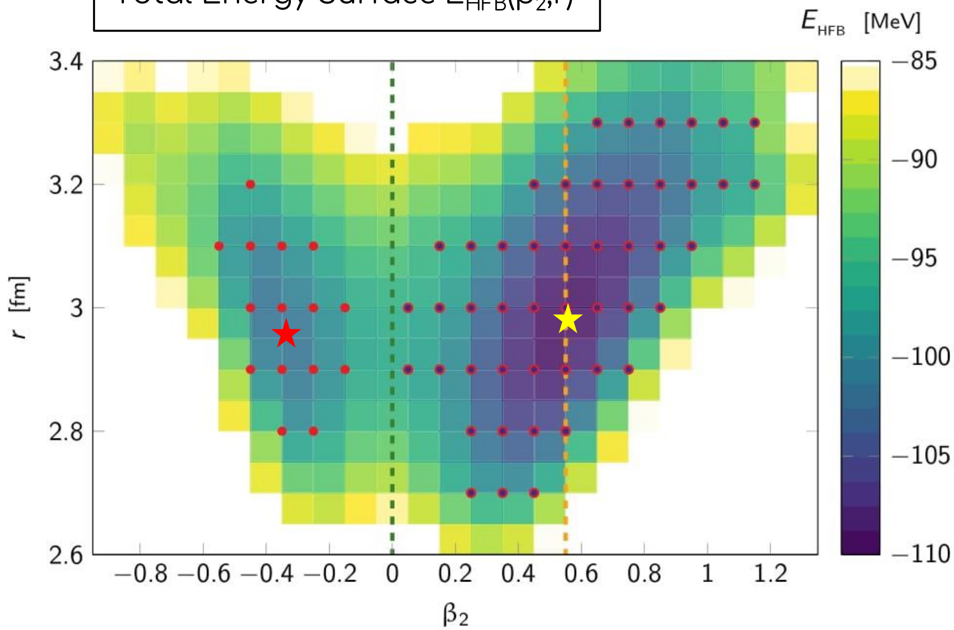

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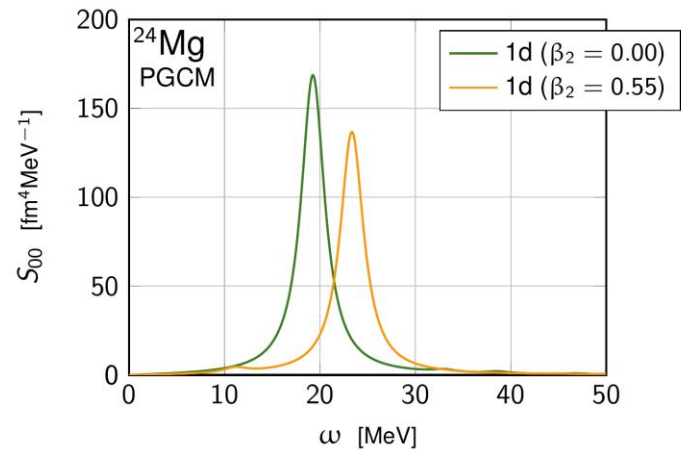

Results

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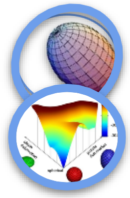
Difficulty



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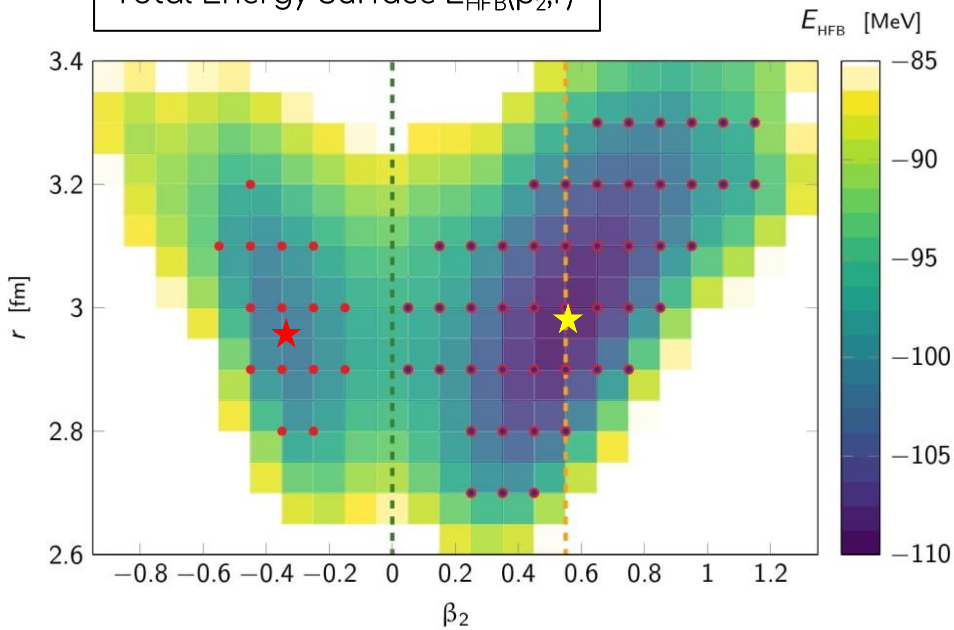

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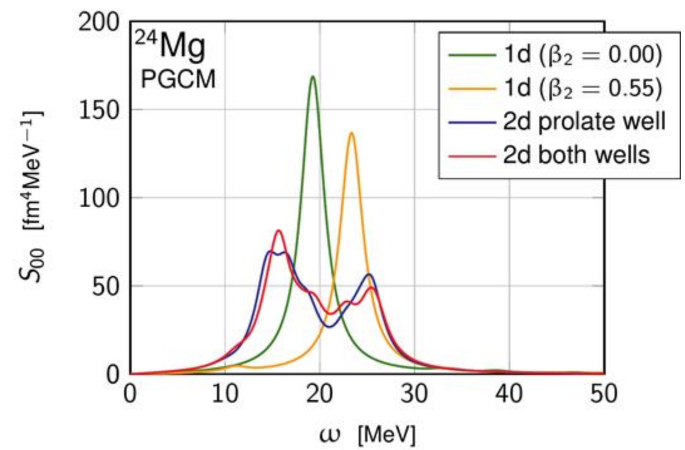
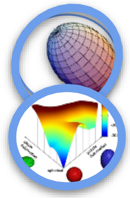


Results

- Dominant prolate minimum
- Important static quadrupole deformation
- No coupling between different wells

Deformation effects in ^{24}Mg

Difficulty



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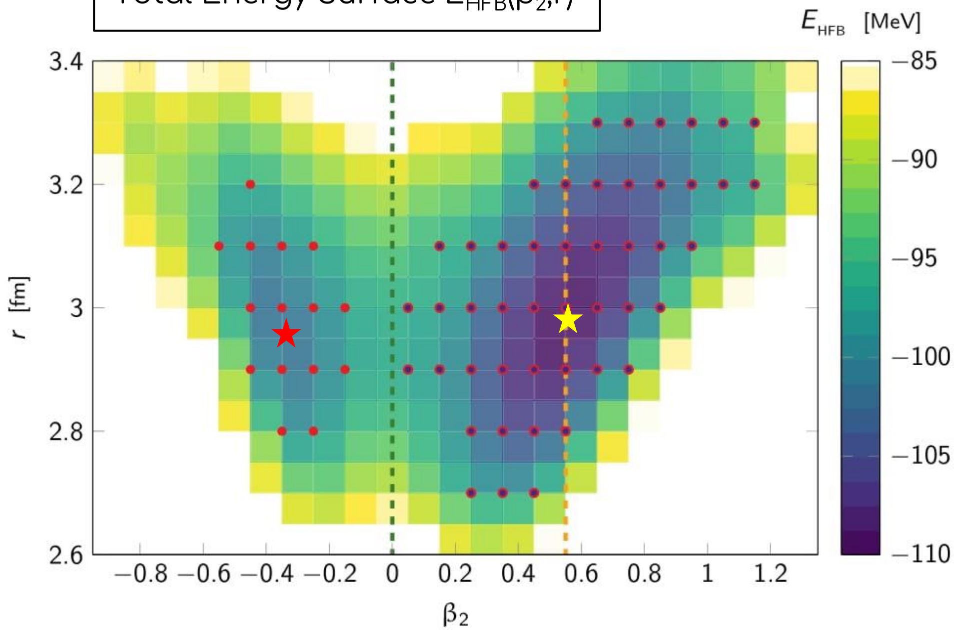

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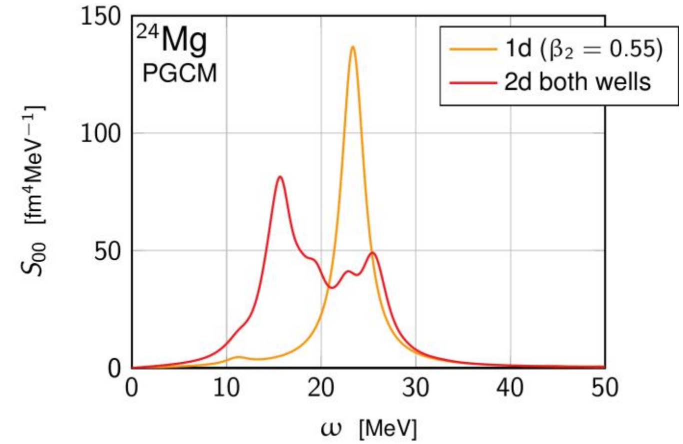
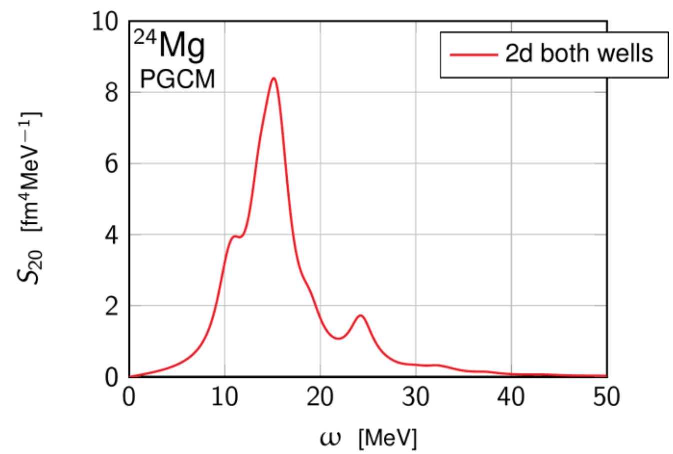


Quadrupole Strength



Results

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- No coupling between different wells
- Coupling to quadrupole fluctuation

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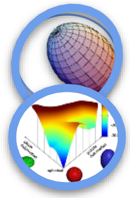
Difficulty



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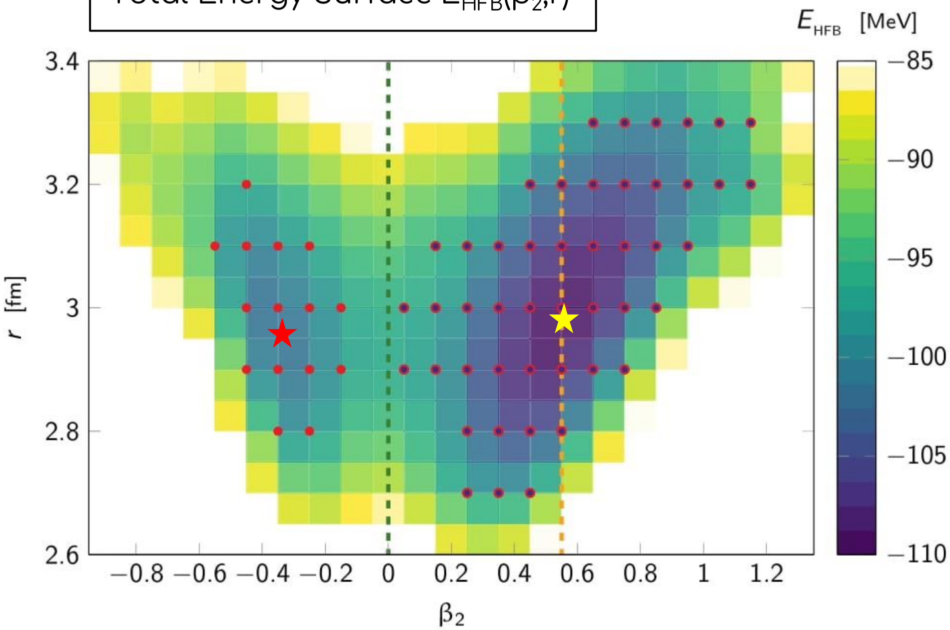

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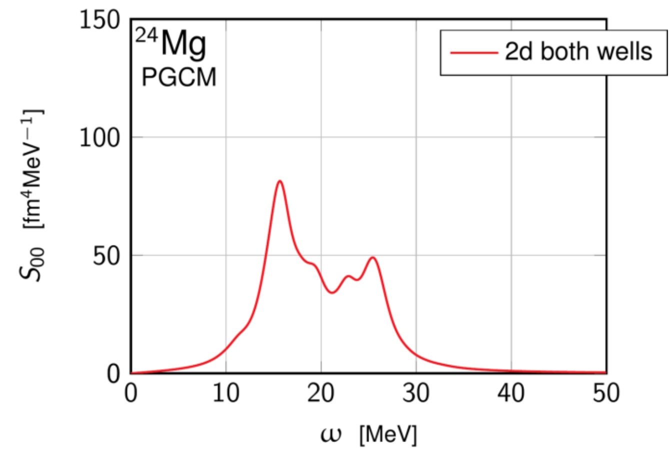
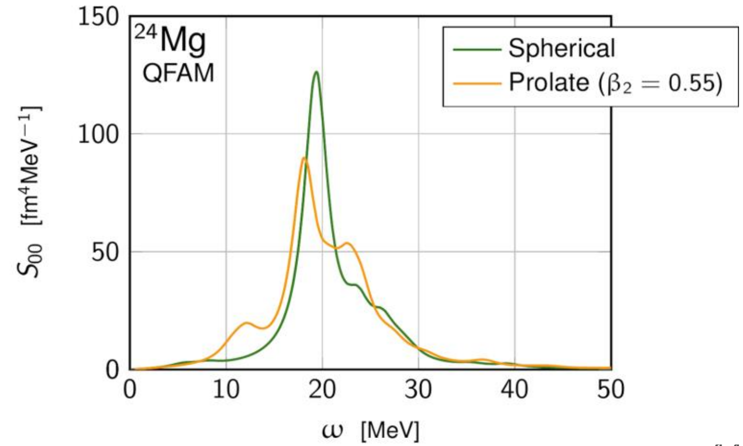


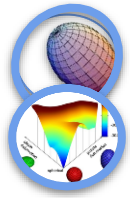


Results

- Dominant prolate minimum
- Important static quadrupole deformation
- No coupling between different wells
- Coupling to quadrupole fluctuation
- Important anharmonic effects QRPA unreliable

Deformation effects in ^{24}Mg

Difficulty

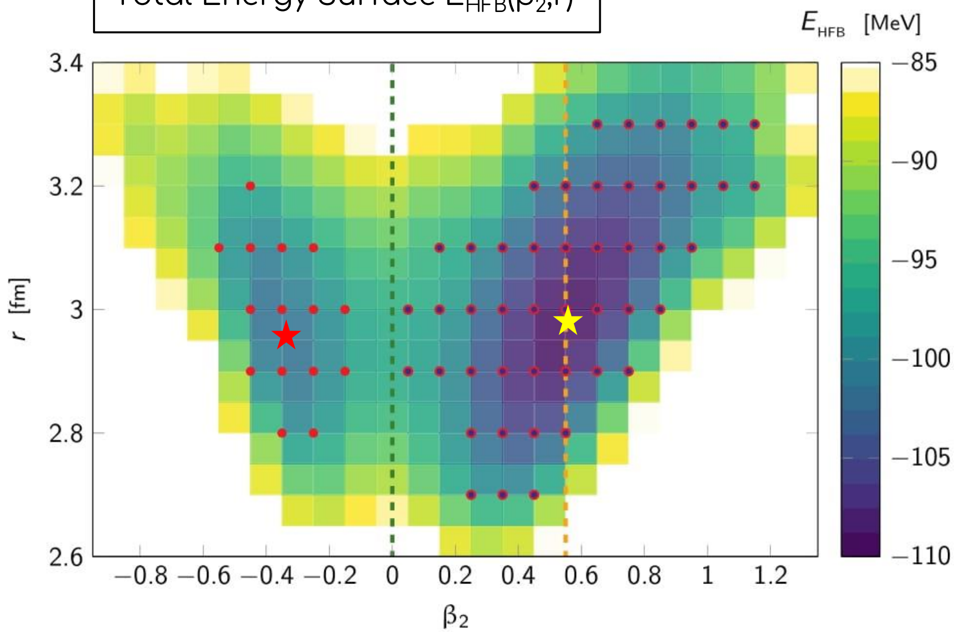


Deformation

Shape coexistence ? (1)

(1) [Dowie et al., 2020]

Total Energy Surface $E_{HFB}(\beta_2, r)$



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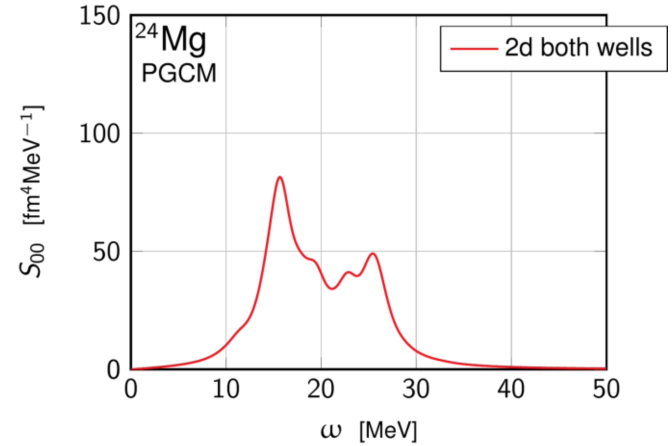

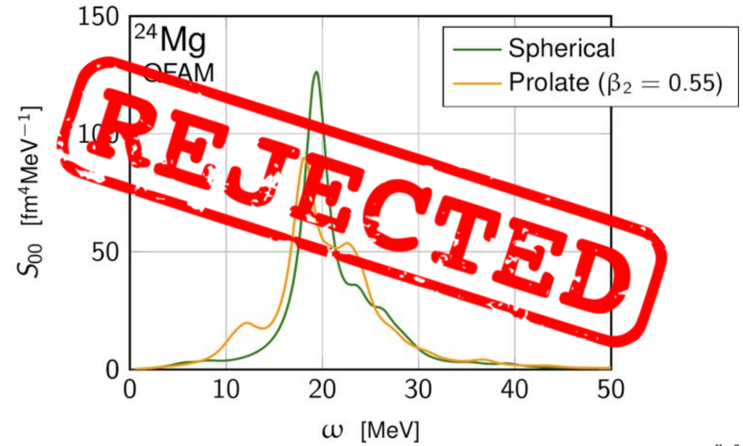

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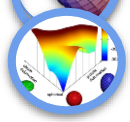

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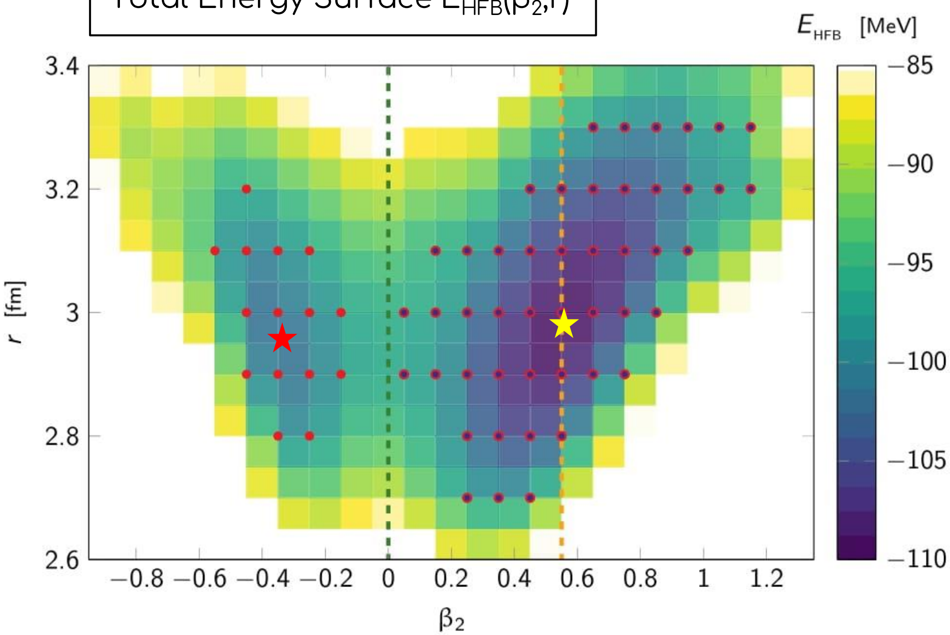

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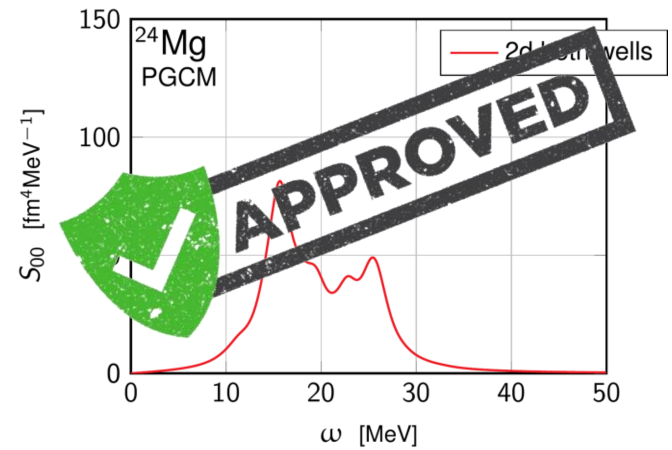

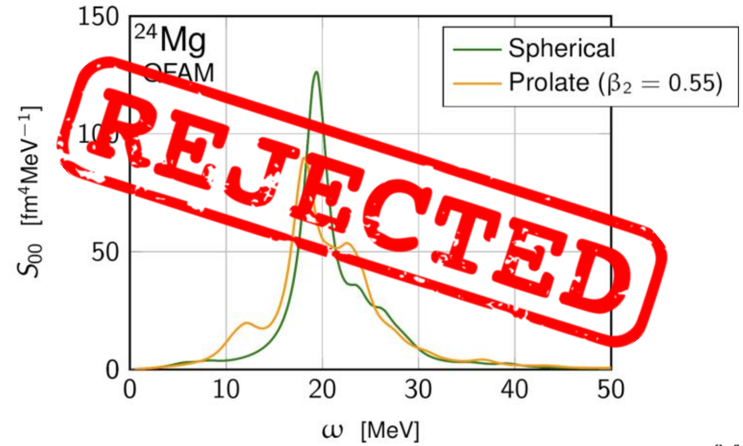

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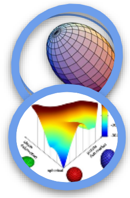

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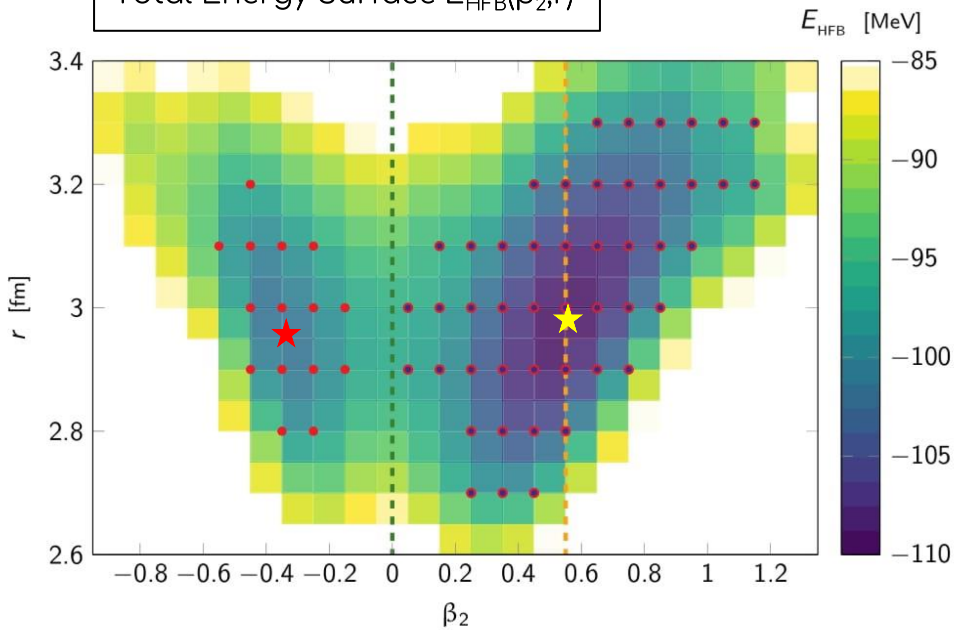

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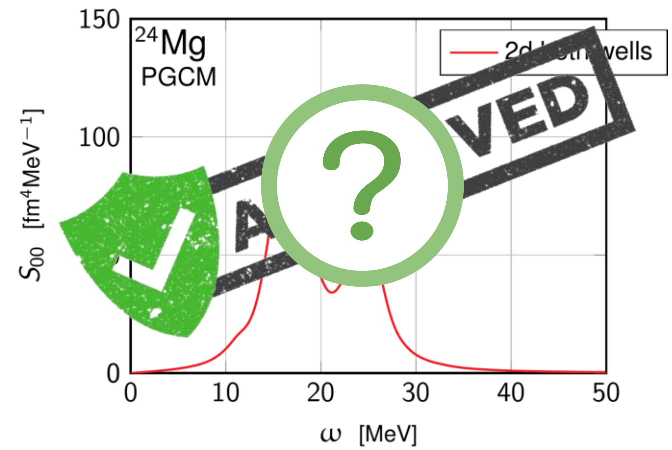

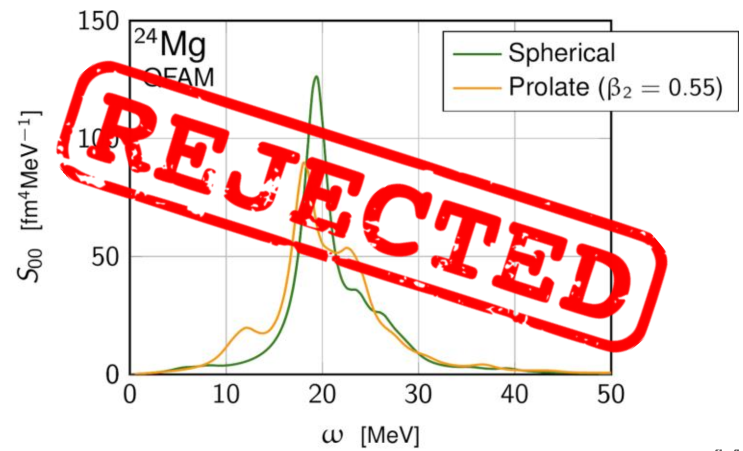

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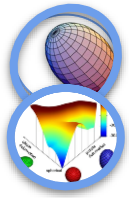

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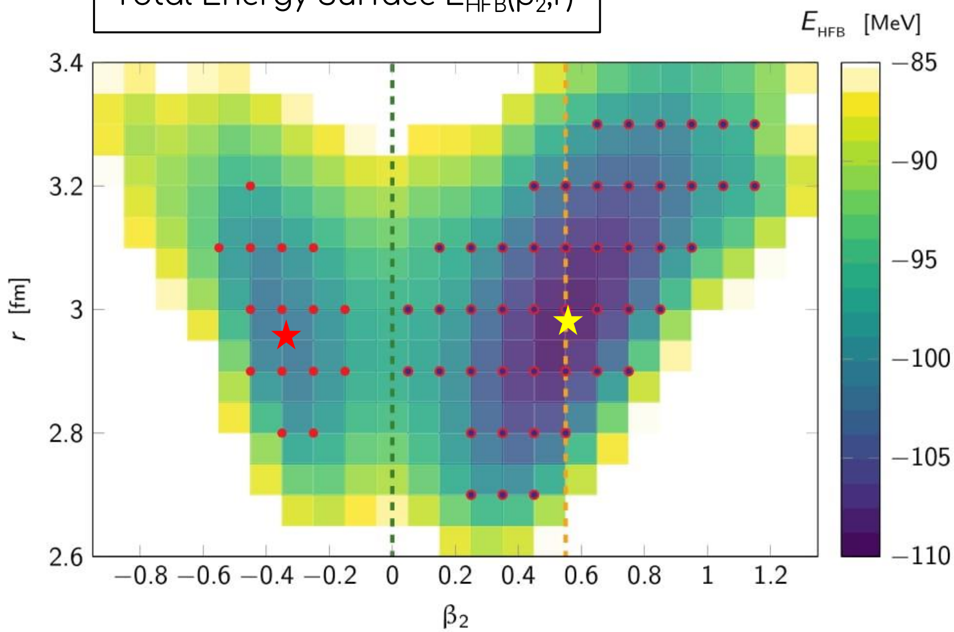

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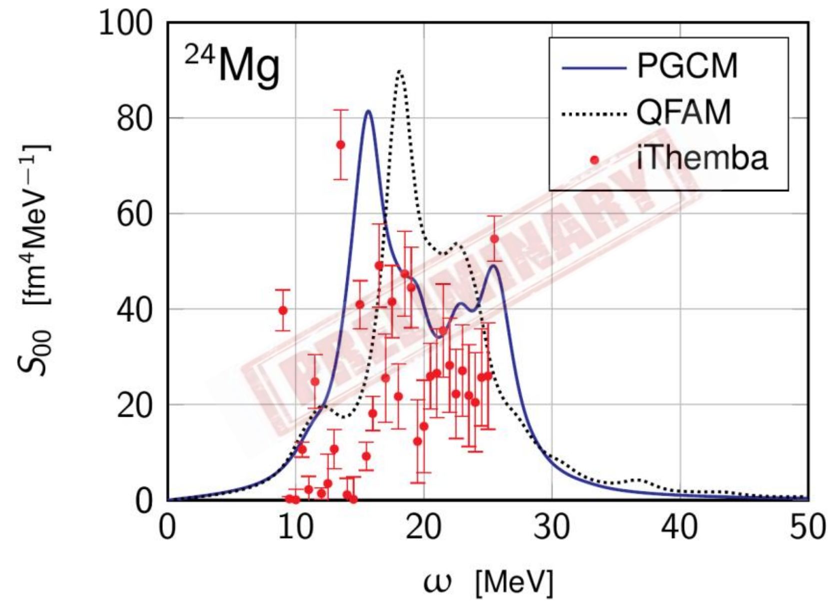

iThemba, Bahini 2021

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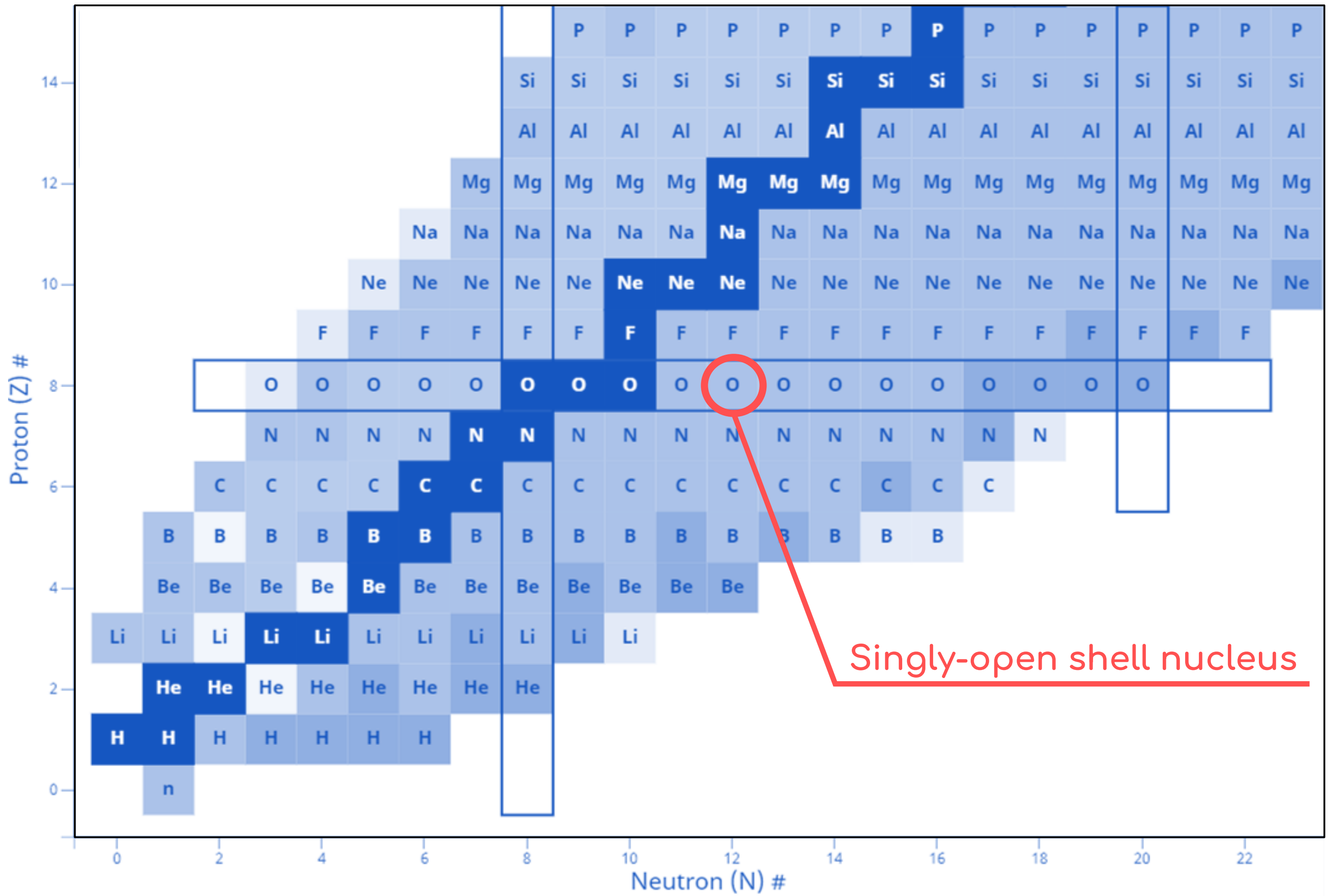
1. **PGCM superior to QRPA**
2. Experiments **useful** and **promising**
3. Data are **not unambiguous**

Superfluidity effects in ^{20}O



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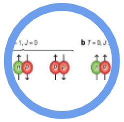

Superfluidity

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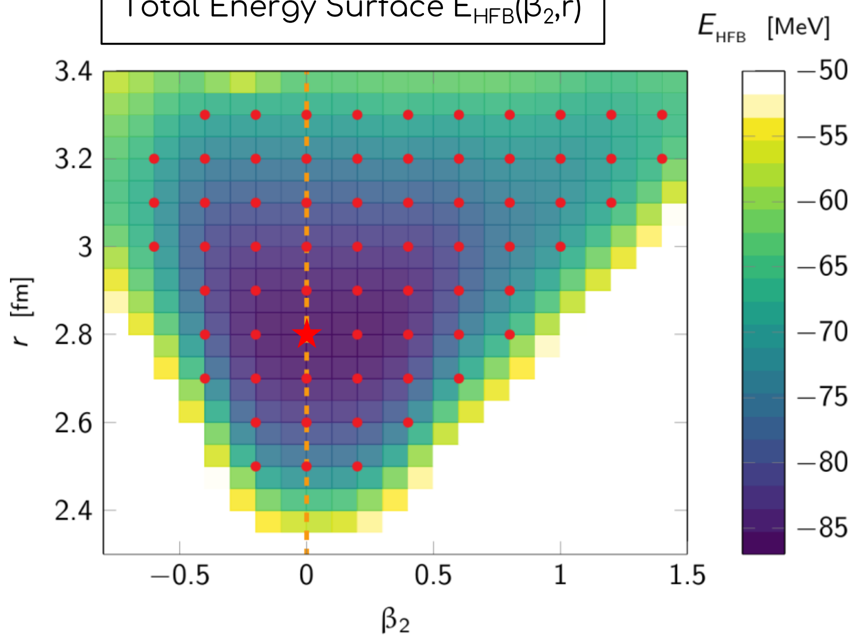

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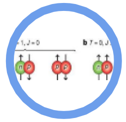

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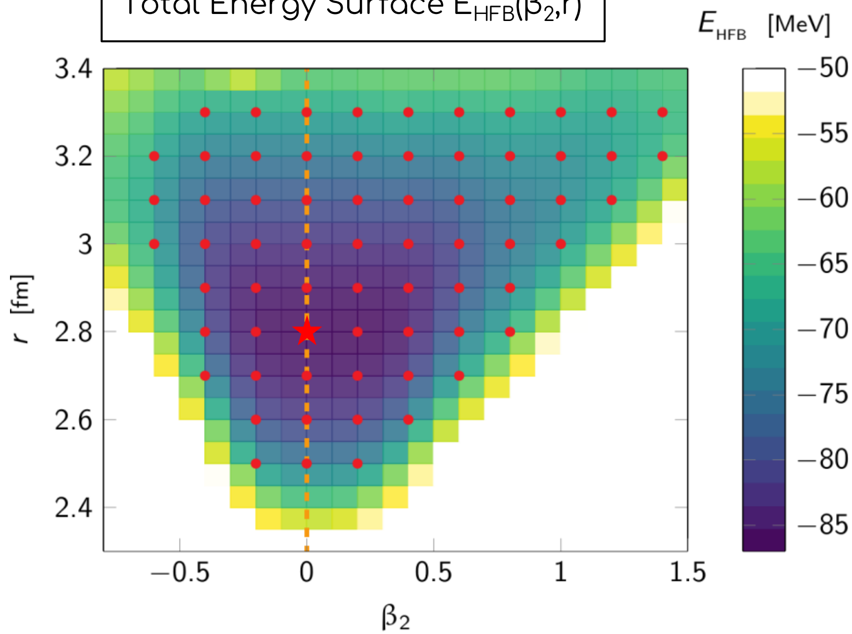

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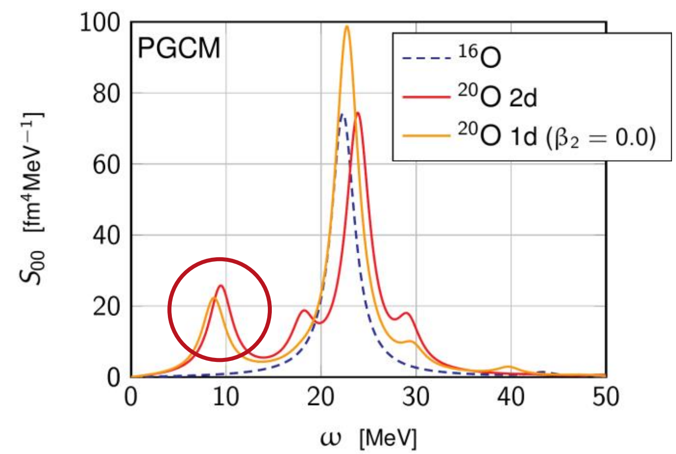

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- In PGCM low energy strength appears

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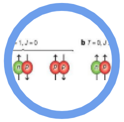

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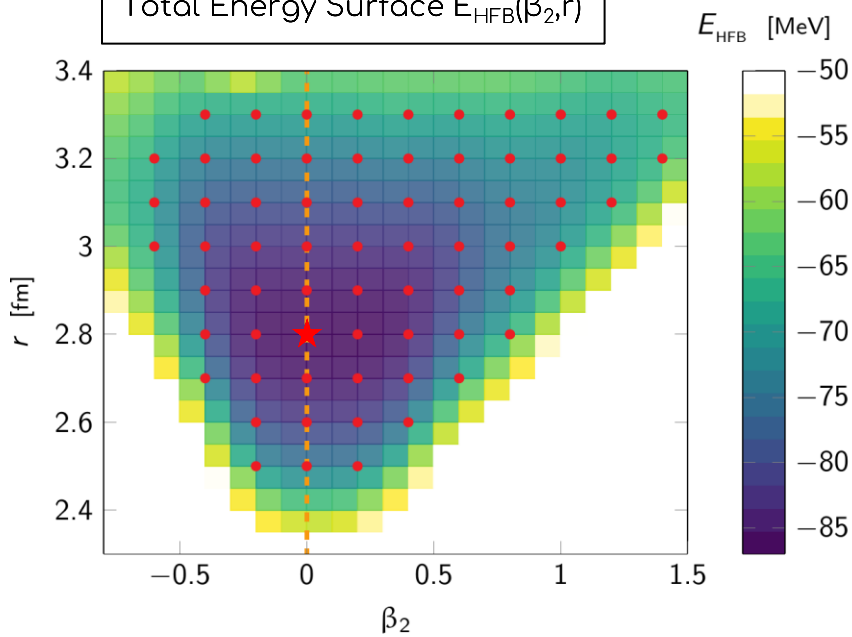

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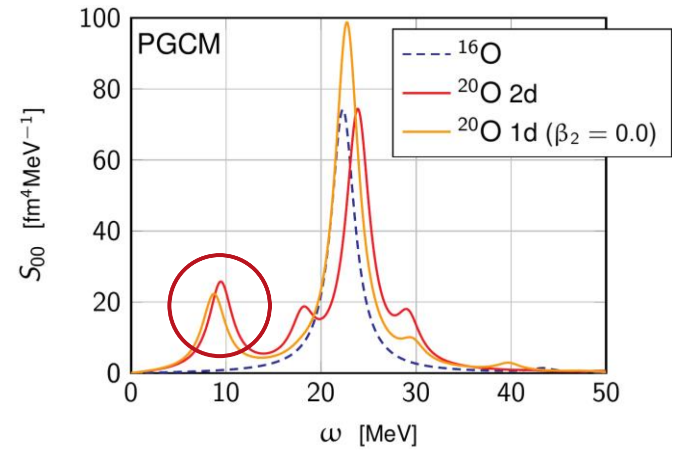

Results

- Single spherical minimum
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- Weak coupling with quadrupolar vibrations

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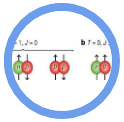

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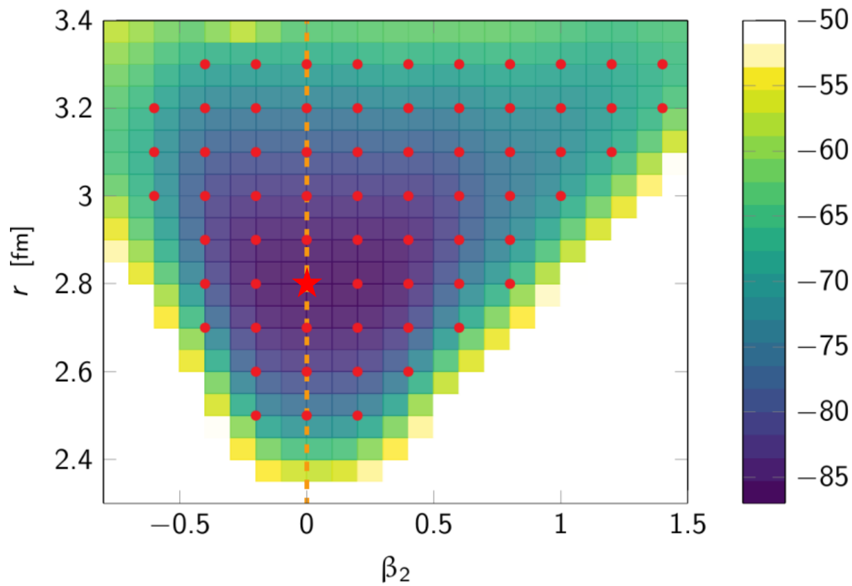

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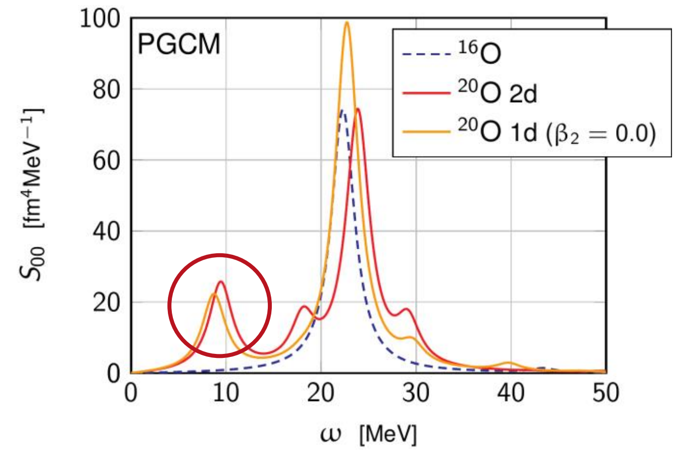

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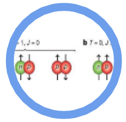

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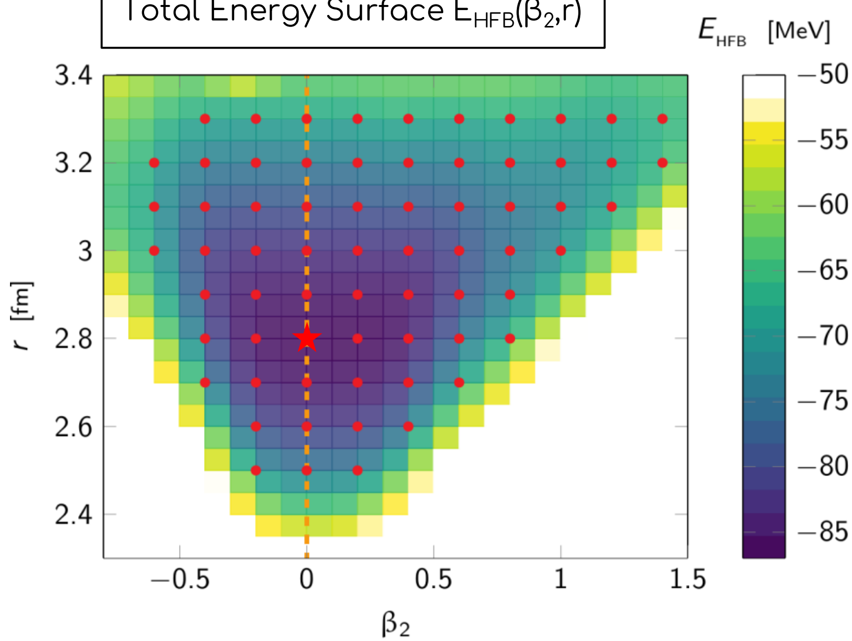

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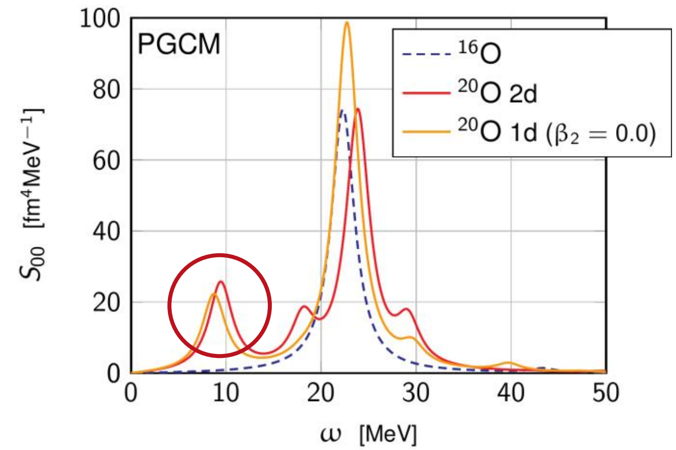

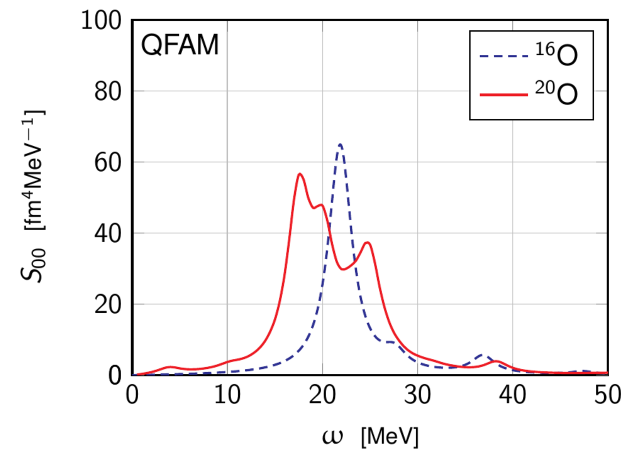

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- Inconsistency between QFAM and PGCM

Outline

- Introduction
- Formalism
- Preliminary results
- **Conclusions**

Conclusions and Perspectives

First ab-initio systematic description of GMR

Choose physics according to selected coordinates

No limitation on the nucleus choice

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Plan of the complete study

- ☐ Static quadrupolar deformation
- ☐ Coupling to quadrupolar vibrations
- ☐ Shape isomers
- ☐ Theoretical comparison of moment computation
- ☐ Hamiltonian uncertainty through different chiral EFT orders
- ☐ Superfluidity (Oxygen isotopic chains, pairing variations)
- ☐ Bubble structure (^{34}Si and ^{36}S)
- ☐ Nuclei of current experimental interest (^{68}Ni and ^{70}Ni) [MSU, AT-TPC]

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Thanks for the attention

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